

VINEET SIR CLASSES
D-1/44, BPTP PARKLANDS, SECTOR 85, FARIDABAD
APPLIED MATHEMATICS
CLASS-XII

ASSIGNMENT ON NUMBERS, QUANTIFICATION, NUMERICAL APPLICATION
AND NUMERICAL INEQUALITIES
(MODULO ARITHMETIC)

1. The value of $14 \pmod{5}$ is
 (a) 4 (b) 2 (c) 3 (d) 1
2. $[(3 \times 7) + 5] \pmod{4}$ is
 (a) 5 (b) 4 (c) 3 (d) 2
3. Today is Tuesday. My uncle will come after 45 days. In which day my uncle will be coming
 (a) Monday (b) Friday (c) Saturday (d) Thursday
4. If $a \equiv 0 \pmod{m}$, then
 (a) $a < m$ (b) $a > m$ (c) a is completely divisible by m (d) m is completely divisible by a
5. $(49 + 58) \pmod{51}$ is
 (a) 4 (b) 5 (c) 6 (d) 7
6. $(15 - 73) \pmod{4}$ is
 (a) 1 (b) 4 (c) 0 (d) 2
7. $(19 \times 121) \pmod{6}$ is
 (a) 2 (b) 1 (c) 3 (d) 4
8. $(18 \times 10) \pmod{7}$ is
 (a) 5 (b) 4 (c) 3 (d) 2
9. $[(194 \times 87) + 45] \pmod{12}$ is
 (a) 1 (b) 4 (c) 3 (d) 2
10. $(-6 \times 5) \pmod{7}$ is
 (a) 1 (b) 5 (c) -5 (d) 2
11. $(32 \times 512) \pmod{4}$ is
 (a) 4 (b) 3 (c) 0 (d) 1
12. (8×14) in 12 h clock is
 (a) 4 o'clock (b) 8 o'clock (c) 6 o'clock (d) 2 o'clock
13. $(09 : 30 + 16 : 40)$ in 24 h clock is
 (a) 03: 10 (b) 26 :10 (c) 2 : 10 (d) 25 : 70
14. The remainder when
 $672 + 541 + 383 + 295 + 101 + 86$ is divided by 3 is
 (a) 2 (b) 1 (c) 0 (d) None of these
15. The remainder when
 $862 \times 783 \times 671 \times 549 \times 411 \times 395 \times 217$ is divided by 8 is
 (a) 5 (b) 6 (c) 4 (d) 3
16. The last two-digit of the product 2345×6789 is
 (a) 35 (b) 25 (c) 05 (d) 15
17. The last three-digits of the product 31234×26875 is
 (a) 234 (b) 750 (c) 760 (d) 850
18. If $100 \equiv x \pmod{7}$, the least positive value of x is
 (a) 3 (b) 1 (c) 2 (d) 4
19. If $150 \equiv 10 \pmod{n}$, then the number of values of n is
 (a) 9 (b) 8 (c) 10 (d) 11
20. The value of x in the set $\{2, 3, 4, 5, 6\}$ satisfying $92 \equiv -3 \pmod{m}$
 (a) 4 (b) 5 (c) 2 (d) 3

ANSWERS

1. (a) 2. (d) 3. (b) 4. (d) 5. (b) 6. (d) 7. (b) 8. (a) 9. (c) 10. (b)
11. (c) 12. (a) 13. (c) 14. (a) 15. (b) 16. (c) 17. (b) 18. (c) 19. (d) 20. (b)

Directions (Q. Nos. 21-26) Each of these questions contains two statements Assertion (A) and Reason (R). Each of the questions has four alternative choices, any one of the which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) A is true, R is true; R is a correct explanation of A.

(b) A is true, R is true; R is not a correct explanation of A.

(c) A is true; R is false

(d) A is false; R is true.

21. Assertion (A) $21 \pmod 2 = 1$

Reason (R) Modulo arithmetic is the arithmetic of remainders. So, it can be expressed as a mod $b = r$, where, a is dividend, b is divisor and r is remainder.

22. Assertion (A) It is currently 8:00 am, then it will be 4 : 00 pm in next 500 h.

Reason (R) $a \equiv b \pmod n$ is that difference of a and b i.e. (a - b) is an integral multiple of n i.e. when a - b is divided by n, remainder is zero.

23. Assertion (A) $(486 + 729) \pmod{12} \equiv 3$

Reason (R)

$(a + b) \pmod n \equiv a \pmod n + b \pmod n$

24. Assertion (A) The remainder when $672 + 541 + 383 + 295 + 101 + 86$ is divided by 3 is 2.

Reason (R) If $a \equiv b \pmod n$ and

$c \equiv d \pmod n$, then

$(a + c) \equiv (b + d) \pmod n$.

25. Assertion (A) $(186 \times 93) \pmod 7 = 2$

Reason (R)

$a \cdot b \pmod n \equiv a \pmod n \cdot b \pmod n$.

26. Assertion (A) The remainder when $783 \times 657 \times 594 \times 432 \times 346 \times 251$ is divided by 5 is 3.

Reason (R) If $a \equiv b \pmod n$ and $c \equiv d \pmod n$, then $ac \equiv bd \pmod n$.

ANSWERS

21. (a) 22. (d) 23. (a) 24. (a) 25. (d) 26. (a)

(SHORT/LONG ANSWER QUESTIONS)

27. Prove that:

(i) $2^{160} \equiv 1 \pmod{17}$ (ii) $3^{200} \equiv 1 \pmod{10}$ (iii) $3^{200} \equiv 9 \pmod{13}$ (iv) $10^{515} \equiv 5 \pmod 7$

28. Find the least non-negative remainder when:

(i) 2^{50} is divided by 7 (ii) 2^{151} is divided by 7 (iii) 2^{12} is divided by 41 (iv) 2^{100} is divided by 5

29. Find the remainder when

(i) $79 \times 101 \times 125$ is divided by 11 (ii) $42 \times 44 \times 46$ is divided by 45

(iii) $127 \times 65 \times 59$ is divided by 7 (iv) $34 \times 78 \times 123 \times 217 \times 365$ is divided by 11.

30. Prove that:

(i) $76 \times 204 \equiv 6 \pmod 7$ (ii) $358 \times 499 \equiv 4 \pmod 8$ (iii) $71 \times 73 \times 75 \equiv 2 \pmod{23}$

31. Find the digit in the unit's place of

(i) 2^{12} (ii) 2^{100} (iii) 3^{13} (iv) 3^{50} (v) 3^{123} (vi) 9^{51} (vii) 9^{100} (viii) 7^{249} (ix) 7^{300} (x) 13^{41}

32. Find the digit in the units place of

(i) $111^{111} + 111!$ (ii) $3^{500} + 35!$ (iii) $9^{51} \times 3^{100}$

33. Find the value of x in the set $\{0, 1, 2, 3, 4, 5\}$ such that $73583 \equiv x \pmod 6$.

34. Solve each of the following linear congruences:

1. $22x \equiv 39 \pmod{15}$ 2. $25x \equiv 15 \pmod{29}$ 3. $31x \equiv 75 \pmod{13}$

4. $17x \equiv 12 \pmod{44}$ 5. $47x \equiv 11 \pmod{249}$

ANSWERS

28. (i) 4 (ii) 2 (iii) 32 (iv) 1 29. (i) 5 (ii) 3 (iii) 6 (iv) 5 31. (i) 6 (ii) 6 (iii) 3 (iv) 9 (v) 7 (vi) 9 (vii) 1 (viii) 7 (ix) 1 (x) 3 32. (i) 1 (ii) 2 (iii) 1 33. 5 34. 1. $x \equiv 12 \pmod{15}$ 2. $x \equiv 18 \pmod{29}$
3. $x \equiv 2 \pmod{13}$ 4. $x \equiv 24 \pmod{44}$ 5. $x \equiv 85 \pmod{249}$

(ALLIGATION AND MIXTURE)

1. 300 g of sugar solution has 40% of sugar in it. The amount of sugar added in the solution to make it 50%
(a) 80 g (b) 60 g (c) 120 g (d) 40 g
2. A grocer mix wheat at Rs. 2.04 per kg and Rs. 2.88 per kg, so as to make a mixture of worth Rs. 2.52 per kg in the ratio
(a) 2 : 3 (b) 3 : 2 (c) 5 : 3 (d) 3 : 4
3. A mixture of certain quantity of milk with 8 L of water is worth 45 paise per litre. If pure milk is worth 54 paise per litre. The quantity of milk in the mixture is
(a) 40 L (b) 35 L (c) 25 L (d) 45 L
4. A merchant has 2000 kg of rice, one part of which he sells at 36% profit and the rest at 16% profit. He gains 28% on the whole. The quantity sold at 16% is
(a) 400 kg (b) 300 kg (c) 900 kg (d) 800 kg
5. A Merchant lent out Rs. 6440 in two parts, one at 8% and the other at 12% interest. If the average interest comes out to be 9%. The amount lent at 12% interest is
(a) Rs. 4830 (b) Rs. 1610 (c) Rs. 1600 (d) Rs. 4850
6. A person has Rs. 8400. He lent a part of it at 4% and the remaining at $3\frac{1}{3}\%$ simple interest. His total annual income was Rs. 294. The sum he lent at 4% is
(a) Rs. 2310 (b) Rs. 2110 (c) Rs.2500 (d) Rs. 2100
7. A merchant has 50 kg of pulse. He sells one part at a profit of 10% and other at 5% loss. Overall he had a gain of 7%. The quantity of pulses, which he sold at 10% profit and 5% loss are
(a) 40 kg, 10 kg (b) 40 kg, 15 kg (c) 40 kg, 12 kg (d) 40 kg, 9 kg
8. A milkman bought 15 L of milk and mixed 3 L of water in it. If the price per kg of the mixture becomes Rs. 22. The cost price of the milk per litre is
(a) Rs. 28.00 (b) Rs. 26.40 (c) Rs. 24.00 (d) Rs. 22.60
9. A butler stole wine from a butt of sherry which contained 80% of spirit and he replaced it by wine containing only 32% spirit. Then, the butt was of 48% strength only. The butt stolen by him is
(a) $\frac{1}{4}$ (b) $\frac{3}{5}$ (c) $\frac{2}{5}$ (d) $\frac{2}{3}$
10. In what proportion, must wheat at Rs. 6.20 per kg be mixed with wheat at Rs. 7.20 per kg, so that the mixture be worth Rs. 6.50 per kg?
(a) 7 : 3 (b) 3 : 7 (c) 4 : 5 (d) 5 : 4
11. A mixture of a certain quantity of milk with 16 L of water is worth Rs. 0.75 per litre. If pure milk is worth Rs. 2.25 per litre, then the amount of milk in the mixture is
(a) 18 L (b) 10 L (c) 8 L (d) 16 L
12. If 50 L of milk solution has 40% milk in it, then amount of milk which should be added to make it 60% in the solution, is
(a) 23 L (b) 24 L (c) 25 L (d) 26 L
13. A vessel of 160 L is filled with milk and water. 70% of milk and 30% of water is taken out of the vessel. It is found that the vessel is vacated by 55%. The quantity of milk and water in this mixture is
(a) Milk = 100 L; Water = 60 L (b) Milk = 50 L; Water = 110 L
(c) Milk =70 L; Water = 90 L (d) Milk = 60 L; Water = 100 L
14. A vessel contains liquids P and Q in the ratio 5 : 3. If 16 L of the mixture are removed and

the same quantity of liquid Q is added, the ratio becomes 3 : 5. The capacity of the vessel is

- (a) 35 L (b) 45 L (c) 40 L (d) 50 L

15. A jar has 40 L milk. From the jar, 8 L of milk was taken out and replaced by an equal quantity of water. If 8 L of the newly formed mixture is taken out of the jar, the final quantity of milk left in the jar is

- (a) 32.5 L (b) 30 L (c) 25.6 L (d) 24.2 L

16. Tea worth Rs. 126 per kg and Rs. 135 per kg are mixed with a third variety in the ratio 1 : 1 : 2. If the mixture is worth Rs. 153 per kg, the price of the third variety per kg will be

- (a) Rs. 169.50 (b) Rs. 170.0 (c) Rs. 175.50 (d) Rs. 180.0

17. Two containers of equal capacity are full of mixture of oil and water. In the first, the ratio of oil to water is 4 : 7 and in the second, it is 7 : 11. Now, both the mixtures are mixed in a bigger container. The resulting ratio of oil to water is

- (a) 149: 247 (b) 247:149 (c) 143 : 241 (d) 241:143

18. A jar contains a mixture of milk and water in the respective ratio of 3 : 1. When 4 L of the mixture is taken out and thereafter 3 L of milk is added to the remaining mixture, the respective ratio of milk and water in the resultant mixture thus formed is 4 : 1. The initial quantity of water in the mixture is

- (a) 1 L (b) 6 L (c) 4 L (d) 2 L

19. A trader mixes 14 kg rice of variety A which costs Rs. 60 per kg with 18 kg of quantity of type B rice. He sells the mixture at Rs. 65 per kg and earns a profit of $\frac{100}{3}$ %. Then, the cost price of type B rice was

- (a) Rs.30 (b) Rs.20 (c) Rs. 40 (d) Rs. 50

20. Jagatram, a milk seller has certain quantity of milk to sell. In what ratio, he should mix water to gain 5% by selling the mixture at the cost price?

- (a) 1 : 10 (b) 1 : 5 (c) 1 : 20 (d) 1 : 15

ANSWERS

1. (b) 2. (d) 3. (a) 4. (d) 5. (b) 6. (d) 7. (a) 8. (b) 9. (d)
10. (a) 11. (c) 12. (c) 13. (a) 14. (c) 15. (c) 16. (c) 17. (a)
18. (c) 19. (c) 20. (c)

Assertion-Reasoning MCQs

Directions (Q.Nos. 21-26) Each of the question contains two statements : Assertion (A) and Reason (R). Each of these questions also has four alternative choices, any one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) A is true, R is true; R is a correct explanation for A.

(b) A is true, R is true; R is not a correct explanation for A.

(c) A is true; R is false.

(d) A is false; R is true.

21. Assertion (A) The ratio of copper and zinc in brass is 13 : 7. In 100 kg of brass, there is 35 kg zinc.

Reason (R)
$$\frac{\text{Quantity of Cheaper}}{\text{Quantity of Dearer}} = \frac{\text{CP of Dearer} - \text{Mean Price}}{\text{Mean Price} - \text{CP of Cheaper}}$$

22. Assertion (A) A shopkeeper mix two types of oranges worth Rs. 30 per kg and Rs. 45 per kg respectively so as to get a mixture at Rs. 40 per kg in the ratio 2:1.

Reason (R) Let the price of cheaper ingredient is denoted by c and price of dearer ingredient is denoted by d and we mix these two types of ingredients in some ratio to produce a mixture at desired price, there

$$\frac{\text{quantity of cheaper ingredient}}{\text{quantity of dearer ingredient}} = \frac{d - m}{m - c}$$

23. Assertion (A) The ratio of milk and water in a mixture of 45 L is 4 : 5.

We added 5 L milk and 10 L water to the mixture, then the ratio of milk and water does not change.

Reason (R) Alligation enables us to find the ratio in which two or more ingredients at the given prices must be mixed to produce a mixture at desired price.

24. Assertion (A) In 30 L mixture of acid, the ratio of acid and water is 2 : 3.

The amount of water should be added to the mixture so that the ratio of acid and water becomes 2 : 5 is 12 L.

Reason (R) Mean price is always lesser than cost price of cheaper quantity and higher than the cost price of dearer quantity.

25. Assertion (A) A mixture of 30 L contains milk and water in the ratio of 7 : 3. The water added to mixture so that the ratio of milk and water becomes 3 : 7 is 40 L.

Reason (R) In the container, milk and water are in the ratio a : b. If x L of water is added to the mixture, the ratio become a : c, then quantity of water in the original mixture is $\frac{bx}{c - a}$.

26. Assertion (A) Mixture of milk and water has been kept in two separate containers. Ratio of milk to water in one of the containers is 5 : 1 and that in the other container is 7 : 2. The mixture of these is added together in the ratio 3 : 2, so that the quantity of milk in the new mixture become 80%.

Reason (R) According to rule of mixture and allegation $\frac{n_1}{n_2} = \frac{A_1 - A_w}{A_w - A_1}$ where, $\frac{n_1}{n_2}$ is the ratio, in which two quantities should be mixed, while A_1 , A_2 and A_w are the cheaper price, dearer price and means price, respectively.

ANSWERS

21. (b) 22. (a) 23. (d) 24. (c) 25. (c) 26. (a)

(SHORT/LONG ANSWER QUESTIONS)

27. In what ratio must a grocer mix two varieties pulses costing Rs. 45 and Rs. 60 per kg respectively so as to get a mixture worth Rs. 49.50 per kg?

28. In what ratio must 25% alcohol be mixed with 50% alcohol to get a mixture of 40% alcohol strength?

29. How many kg of salt at Rs. 10.50 per kg must a man mix with 25 kg of salt at Rs. 6 per kg so that he may, on selling the mixture at Rs. 10 per kg, gain 25% on the outlay?

30. Five litres of water is added to a certain quantity of pure milk costing Rs. 60 per litre. If by selling the mixture at the same price as before, a profit of 20% is made, what is the amount of pure milk in the mixture?

31. Alcohol costs Rs. 35 per litre and Kerosene oil costs Rs. 25 per litre. In what proportion should these be mixed so that the resulting mixture may be Rs. 27.50 per litre?

32. A cup of milk contains 3 parts of pure milk and 1 part of water. How much mixture must be withdrawn and water substituted in order that resulting mixture may be half milk and half water?

33. A mixture contains milk and water in the ratio 3 : 2. If 4 litres of water is added to the mixture, milk and water in the mixture become equal. Find the quantity of milk in the mixture, in litres.

34. A mixture contains milk and water in the ratio 8 : x. When 33 litres of mixture and 3 litres of water are mixed, the ratio of milk and water becomes 2 : 1 find the value of x.

35. A vessel contains 56 litres of mixture of milk and water in the ratio 5 : 2. How much water should be mixed with it so that milk to water ratio becomes 4 : 5?

36. How many litres of water should be added to a 30 litre mixture of milk and water containing

- milk and water in the ratio 7 : 3 such that the resultant mixture has 40% of water in it?
37. The ratio of milk and water in the mixture of water and milk is 4 : 3. If 6 litres of water is added to this mixture, the ratio of milk and water becomes 8 : 7. What is the quantity of milk in the original mixture?
38. 35 kg of types sandal powder, which costs Rs. 614 per kg, was mixed with a certain amount of type S_2 sandal powder, which costs Rs. 695 per kg. Then the mixture was sold at the rate of Rs. 767 per kg and 18% profit was earned. What was the amount (in kg) of type S_2 sandal powder in the mixture?

ANSWERS

27. 7 : 3 28. 2 : 3 29. 20 kg 30. 25 litres 31. 1 : 3 32. $\left(\frac{1}{3}\right)^{\text{rd}}$ 33. 20 litres 34. 3
35. 34 litres 36. 5 litres 37. 48 litres 38. 28 kg

BOATS AND STREAMS

- A man can row with a speed of 6 km/h in still water. If the speed of stream is 2 km/h, his speed with the stream will be
(a) 2 km/h (b) 4 km/h (c) 6 km/h (d) 8 km/h
- If the speed of a boat in still water is 8 km/h and the rate of stream is 4 km/h, then upstream speed of the boat is
(a) 2 km/h (b) 4 km/h (c) 6 km/h (d) 8 km/h
- If the speed of a swimmer in still water is 9 km/h. Then, the downstream speed of the swimmer, when the river is flowing with the speed of 6 km/h, is
(a) 15 km/h (b) 18 km/h (c) 3 km/h (d) 12 km/h
- Shantanu can row upstream at 10 km/h and downstream at 18 km/h. Shantanu's rate in still water will be
(a) 7 km/h (b) 14 km/h (c) 8 km/h (d) 4 km/h
- A boat goes 48 km downstream in 20 h. It takes 4 h more to cover the same distance against the stream. The speed of the boat in still water is
(a) 2.2 km/h (b) 2 km/h (c) 4 km/h (d) 4.2 km/h
- Ravi can row upstream 8 km/h and downstream 16 km/h. The rate of current is
(a) 2 km/h (b) 3 km/h (c) 4 km/h (d) 5 km/h
- A person can row with the stream at 8 km/h and against the stream at 6 km/h. The speed of the current is
(a) 1 km/h (b) 2 km/h (c) 4 km/h (d) 5 km/h
- A boatman rows 1 km in 5 min along the stream and 6 km in 1 h against the stream. The speed of the stream is
(a) 3 km/h (b) 6 km/h (c) 10 km/h (d) 12 km/h
- The speed of a boat in still water is 500% more than the speed of the current. The respective ratio between the speed of the boat downstream and speed of the boat upstream is
(a) 9:2 (b) 7 : 3 (c) 7 : 5 (d) 9 : 4
- Rajnish can row 12 km/h in still water. It takes him twice as long to row up as to row down the river. The rate of stream will be
(a) 1 km/h (b) 2 km/h (c) 3 km/h (d) 4 km/h
- Ashutosh can row 24 km/h in still water. It takes him twice as long as to row up as to row down the river. The rate of stream is
(a) 4 km/h (b) 18 km/h (c) 8 km/h (d) 15 km/h
- Kamal can row a certain distance downstream in 12 h and can return the same distance in 18 h. If the stream flows at the rate of 6 km/h, then the speed of Kamal in still water is
(a) 10 km/h (b) 20 km/h (c) 30 km/h (d) 40 km/h
- Kainal can row a certain distance downstream in 12 h and can return the same distance in

- 18 h. If the speed of Kamal in still water is 12 km/h, then the speed of the stream will be
 (a) 1.2 Km/h (b) 2.A km/h (c) 3.6 km/h (d) 4.8 km/h
14. A person can row downstream 20 km in 2 h and upstream 4 km in 2 h. The speed of the current is
 (a) 2 km/h (b) 2.5 km/h (c) 3 km/h (d) A km/h
15. Ramesh rows in still water with a speed of 4.5 km/h to go to a certain place and to come back. Then, his average speed for the whole journey, if the river is flowing with a speed of 1.5 km/h, is
 (a) 8 km/h (b) 6 km/h (c) A km/h (d) 2 km/h
16. A man can row at 10 km/h in still water. If he takes total 5 h to go to a place 24 km away and return, then the speed of the water current is
 (a) 2 km/h (b) 3 km/h (c) $\frac{1}{2}$ km/h (d) 1 km/h
17. A man can row 12'km/h in still water. When the river is running at 2.4 km/h, it takes him 1 h to row to a place and to come back. How far is the place?
 (a) 5.76 km (b) A.76 km (c) 5.26 km (d) 4.26 km
18. A boat's speed in still water is 10 km/h, while river is flowing with a speed of 2 km/h and time taken to cover a . certain distance upstream is 4 h more than time taken to cover the same distance downstream. The distance is
 (a) 95 km (b) 96 km (c) 100 km (d) 101 km
19. A boat can travel 12.6 km downstream in 35 min. If the speed of the water current is one-fourth the speed of boat in the still water, then what distance the boat can travel upstream in 28 min?
 (a) 6 km (b) 7.5 km (c) 8.5 km (d) 8 km
20. Pawan can row 24 km/h in still water. When the river is running at 4.8 km/h, it takes him 1 h to row to a place and to come back. How far is the place?
 (a) 11.52 km (b) 1A km (c) 12.52 km (d) 15 km

ANSWERS

1. (d) 2. (b) 3. (a) 4. (b) 5. (a) 6. (c) 7. (a) 8. (a) 9. (c) 10. (d)
 11. (c) 12. (c) 13. (b) 14. (d) 15. (c) 16. (a) 17. (a) 18. (b) 19. (a) 20. (a)

Assertion-Reasoning MCQs

Directions (Q. Nos. 21-26) Each of these questions contains two statements Assertion (A) and Reason (R). Each of the questions has four alternative choices, any one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

- (a) A is true, R is true; R is a correct explanation of A.
 (b) A is true, R is true; R is not a correct explanation of A.
 (c) A is true; R is false
 (d) A is false; R is true.

21. Assertion (A) Rohan can row with a speed of 16 km/h in still water. If the speed of stream is 12 km/h, then speed of Rohan with stream will be 26 km/h.

Reason (R) If the speed of a boat in still water is x km/h and speed of the stream is y km/h, then speed downstream will be x + y.

22. Assertion (A) A boat takes 4 h to row 40 km downstream and 8 h to row the same distance upstream, Then, the boats rate in still water is $\frac{15}{2}$ km/h.

Reason (R) The rate of boat in still water is given by $\frac{(\text{Speed downstream} + \text{Speed upstream})}{2}$

23. Assertion (A) A boat running downstream course a distance of 18 km in 3 h while for covering the same distance upstream, it takes 6 h. Then, the speed of the boat in still water will

be $\frac{9}{2}$ km/h.

Reason (R) Speed of the boat in still water is given by $\frac{1}{2}$ (rate downstream + rate upstream).

24. Assertion (A) A person swim in water with a speed of 19 km/h in still water. If the speed of the stream is 5 km/h, then the time taken by the person to go 120 km downstream is 5 h.

Reason (R) Time taken to travel d km downstream = $\frac{\text{distance (d)}}{\text{downstream speed}}$

25. Assertion (A) In one hour, a boat goes 14 km/h in the direction of the stream and 8 km/h against the direction of the stream. Then, the speed of the boat in still water is 11 km/h.

Reason (R) If the speed of a boat in still water is 10 km/h and the rate of stream is 6 km/h, then the upstream speed of the boat is 4 km/h.

26. Assertion (A) Speed of a boat in standing water is 10 km/h and the speed of the stream is 2.5 km/h. A man rows to a place at a distance of 125 km and comes back to the starting point. Then, the total time taken by him is 26.67 h.

Reason (R) A person can row with the stream at 6 km/h and against the stream at 4 km/h. The speed of the current is 1 km/h.

ANSWERS

21. (d) 22. (a) 23. (a) 24. (a) 25. (b) 26. (b)

(SHORT/LONG ANSWER QUESTIONS)

27. The speed of a boat when rowing downstream is 32 km/hr, whereas when rowing upstream it is 24 km/hr. Find the speed of the boat in still water and speed of the stream.

28. A man can row a boat 2 km against the stream in 20 minutes and returns back at the same point in 15 minutes. Find the rate of rowing the boat and speed of the current of the river.

29. A man takes 3 hours 45 minutes to row a boat 15 km downstream of a river and 2 hours 30 minutes to cover a distance of 5 km upstream. Find the rate of rowing the boat and the speed of river current.

30. A man can row a boat at 6 km/hr in still water. If the current of water in the river is at 2 km/hr, it takes him 3 hours to row to a place and come back. How far is the place?

31. A man can row a boat at 7.5 km/hr in still water. If in a river flowing at 1.5 km/hr, it takes him 50 minutes to row to a place and back, how far off is the place?

32. A man can row a boat in still water at the rate of 6 km/hr. If the water in a river flows at the rate of 2 km/hr, it takes 3 hours more in upstream than to go downstream for the same distance. How far is the place?

33. A man rows a boat 10 km upstream in a river and returns back to the starting point in 55 minutes. If the water in the river is flowing at the speed of 2 km/hr, find the rate at which the man can row the boat in still water.

34. A boat goes 8 km upstream and then returns. Total time taken is 4 hours 16 minutes. If the speed of the current is 1 km/hr, find the actual speed of the boat.

35. A man can row 6 km/hr in still water. It takes him twice as long to row up as to row down the river. Find the rate of the stream.

36. A man can row $9\frac{1}{3}$ km/hr in still water and finds that it takes him thrice as much time to row up than as to row down the same distance in the river. Find the speed of the current.

37. The speed of a boat in still water is 5 times that of the current, it takes 1.1 hours to row to a point Q from point P downstream. The distance between points P and Q is 13.2 km. How much distance will it cover in $5\frac{1}{5}$ hours upstream?

38. A boat can travel 36 km upstream in 5 hours. If the speed of the stream is 2.4 km/hr, how much time will the boat take to cover a distance of 78 km downstream?

39. A man can row 40 km upstream and 55 km downstream in 13 hours. Also, he can row 30

km upstream and 44 km downstream in 10 hours. Find the speed of the man in still water and speed of the current.

40. A man rows his boat a place 48 km distant in 14 hours. He finds that he can row 4 km with the stream in the same time as 3 km against the stream. Find the rate of stream.

41. A boatman takes 2 hours for travelling downstream from a point A to a point B coming back to point A. If $AB = 6$ km and the water flows in the river at 4 km/hr, find the speed of the boatman in still water.

42. A man can row $\frac{3}{4}$ of a km against the stream in $11\frac{1}{4}$ minutes and returns in $7\frac{1}{2}$ minutes. Find the speed of the man in still water.

43. A man rows to a place 48 km distant and back in 14 hours. He finds that he can row 4 km with the stream in the same time as 3 km against the stream. Find the rate of stream.

44. A man takes 2.2 times as long to row a distance upstream as to row the same distance downstream. If he can row 55 km downstream in 2 hours 30 minutes, what is the speed of the boat in still water?

45. A boat covers 24 km upstream and 36 km downstream in 6 hours while it covers 36 km upstream and 24 km downstream in $6\frac{1}{2}$ hours. Find the speed of boat in still water and the speed of the steam.

ANSWERS

27. 28 km/hr, 4 km/hr 28. 7 km/hr, 1 km/hr 29. 3 km/hr, 1 km/hr 30. 8 km 31. 3 km 32. 24 km 33. 22 km/hr 34. 4 km/hr 35. 2 km/hr 36. $\frac{14}{3}$ km/hr 37. 41.6 km 38. 6 hour 30 minutes 39. 8 km/hr, 3 km/hr 40. 1 km/hr 41. 8 km/hr 42. 5 km/hr 43. 1 km/hr 44. 16 km/hr 45. 10 km/hr, 2 km/hr

PIPES AND CISTERNS

1. Through an inlet, a tank takes 8 h to get filled up. Due to a leak in the bottom, it takes 2 h more to get it filled completely. If the tank is full. The time will the leak take to empty it is

(a) 16 h (b) 20 h (c) 32 h (d) 40 h

2. A cistern can be filled up in 4 h by an inlet A. An outlet B can empty the cistern in 8 h. If both A and B are opened simultaneously, then the time will the cistern get filled is

(a) 15 h (b) 7 h (c) 8 h (d) 6 h

3. A tap can fill an empty tank in 12 h and a leakage can empty the tank in 20 h.

If tap and leakage both work together, then the time taken to fill the tank is

(a) 25 h (b) 40 h (c) 30 h (d) 35 h

4. A pipe can fill a tank in 20 h. Due to a leak in the bottom, it is filled in 40 h.

If the tank is full, then the time taken by leak to empty it is

(a) 40 h (b) 30 h (c) 50 h (d) 30 h

5. A tank (of capacity 160 L) has one inlet, A and one outlet, B. Inlet A, alone can fill the empty tank in 6 h and outlet B alone can empty the full tank in 24 h. The time taken by inlet A to fill 16 L water in the tank when outlet B is also open is

(a) 1 (b) $\frac{4}{5}$ (c) $\frac{11}{5}$ (d) $1\frac{1}{2}$

6. If a pipe can fill a tank in 2 h and another pipe can fill the same tank in 6 h, then the part of a tank will be filled by both the pipes in 1 h, if they are opened simultaneously is

(a) $\frac{2}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{3}{4}$

7. If a pipe can fill a tank in 5 h and another pipe can empty the tank in 10 h, then the part of a tank will be filled by both pipes in 1 h, if both pipes are open simultaneously is

(a) $\frac{1}{10}$ (b) $\frac{2}{5}$ (c) $\frac{2}{3}$ (d) $\frac{1}{4}$

8. Two pipes X and Y can fill a cistern in 6 min and 7 min, respectively. Starting with pipe X, both the pipes are opened alternately, each for 1 min. The time taken by both to fill the cistern

is

- (a) $6\frac{2}{7}$ min (b) $6\frac{3}{7}$ min (c) $6\frac{5}{7}$ min (d) $6\frac{1}{7}$ min

9. A tank can be filled by pipe X in 2 h and pipe Y in 6 h. At 10 am, pipe X was opened. The time taken to fill the tank, if pipe Y is opened at 11 am is

- (a) 12 : 45 pm (b) 5 : 00 pm (c) 11: 45 am (d) 11: 50 am

10. Three pipes m, n and p can fill a tank separately in 4, 5 and 10 h, respectively. The time taken by all the three pipes to fill the tank when the pipes are opened together is

- (a) $2\frac{9}{11}$ h (b) 1 h (c) $1\frac{9}{11}$ h (d) 2 h

11. Pipe A can fill a tank in 30 min, while pipe B can fill the same tank in 10 min and pipe C can empty the full tank in 40 min. If all the pipes are opened together, then the time taken to make the tank full is

- (a) $9\frac{3}{13}$ h (b) $9\frac{4}{13}$ h (c) $9\frac{7}{13}$ h (d) $9\frac{9}{13}$ h

12. Pipes A and B can fill a tank in 5 h and 6 h, respectively. Pipe C can fill it in 30 h. If all the three pipes are opened together, then the time taken to fill the tank is

- (a) $3\frac{3}{14}$ h (b) $2\frac{1}{2}$ h (c) $3\frac{9}{14}$ h (d) $2\frac{1}{14}$ h

13. Three taps A, B and C together can fill an empty cistern in 10 min. The tap A alone can fill it in 30 min and the tap B alone can fill it in 40 min. The tap C alone fill it in

- (a) 16 min (b) 24 min (c) 32 min (d) 40 min

14. A, B and C are three pipes connected to a tank. A and B together fill the tank in 6 h, B and C together fill the tank in 10 h and A and C together fill the tank in 12 h. The time taken by A, B and C to fill the tank together is

- (a) 9 h (b) $5\frac{3}{7}$ h (c) $5\frac{2}{7}$ h (d) $5\frac{5}{7}$ h

15. Two pipes P and can fill a cistern in 12 min and 15 min, respectively. If both pipes are opened together and at the end of 3 min, the first is closed, then the time taken to fill the cistern is

- (a) $8\frac{1}{4}$ min (b) $8\frac{3}{4}$ min (c) 5 min (d) $8\frac{1}{2}$ min

16. Pipe A alone can fill a tank in 8 h. Pipe B alone can fill the same tank in 10 h. If both the pipes are opened but after 2 h, pipe A is closed, then pipe B will fill the tank in

- (a) 6 h (b) $3\frac{1}{2}$ h (c) 4 h (d) $5\frac{1}{2}$ h

17. Two pipes A and B can fill a tank in 24 min and 32 min, respectively. If both the pipes are opened together, then after how much time pipe B should be closed so that the tank is full in 9 min?

- (a) 40 min (b) 30 min (c) 10 min (d) 20 min

18. Two pipes A and B are opened together to fill a tank. Both the pipes fill the tank in time t. If A separately takes 4 min more time than t to fill the tank and B takes 64 min more time than t to fill the tank, then the value of t is

- (a) 14 min (b) 12 min (c) 10 min (d) 16 min

19. Two pipes A and B are opened together to fill a tank. Both pipes fill the tank in a certain time. If A separately takes 16 min more than the time taken by (A + B) and B takes 9 min more than the time taken by (A + B). Then, time taken by A and B to fill the tank when both the pipes are opened together is

- (a) 10 min (b) 12 min (c) 15 min (d) 8 min

20. A full tank gets emptied in 6 min due to presence of an orifice in it. On opening a tap which can fill the tank at the rate of 8 L/min, the tank gets emptied in 10 min. The capacity of tank is

- (a) 80 L (b) 90 L (c) 120 L (d) 100 L

21. A tank has a leak which would empty it in 8 h. A tap is turned on which admits 3 L a min

into the tank and it is now emptied in 12 h. The capacity of tank is

- (a) 4320 (b) 4000 (c) 2250 (d) 4120

22. A full tank get emptied in 30 min due to leakage. If a tap which fills it at a rate of 10 L/min is opened, then it get emptied in 50 min. The capacity of tank is

- (a) 720 L (b) 750 L (c) 710 L (d) 760 L

23. Pipe A is 3 times slower than the pipe B and takes 16 min more time than the pipe B. The time taken by pipe B to fill the tank is

- (a) 10 min (b) 7 min (c) 6 min (d) 8 min

24. Pipe A is 4 times faster than pipe B and takes 30 min less than the pipe B to fill the tank. If both pipes are opened together, then the cistern will be fill in

- (a) 6 min (b) 8 min (c) 12 min (d) 9 min

25. If tap A can fill a tank 2 times faster than tap B and takes 20 min less than tap B to fill the tank. If both the taps are opened simultaneously, then the time taken to fill the tank by tap A is

- (a) 20 min (b) 25 min (c) 15 min (d) None of these

26. Tap A can fill a tank 3 times faster than tap B and takes 28 min less than tap B to fill the tank. If both the taps are opened simultaneously, then the time taken to fill the tank is

- (a) 21 min (b) $\frac{21}{2}$ min (c) 22 min (d) None of these

27. Pipe A is 2 times slower than pipe B and takes 9 min more time than the pipe B. How much time will pipe A take to fill the tank?

- (a) 18 min (b) 12 min (c) 10 min (d) 16 min

ANSWERS

1. (d) 2. (c) 3. (c) 4. (a) 5. (b) 6. (a) 7. (a) 8. (b) 9. (c)
 10. (c) 11. (a) 12. (b) 13. (b) 14. (d) 15. (a) 16. (d) 17. (d)
 18. (d) 19. (b) 20. (c) 21. (a) 22. (b) 23. (d) 24. (b)
 25. (a) 26. (b) 27. (a)

Assertion-Reasoning MCQs

Directions (Q.Nos. 28-34) Each of these question contains two statements. Assertion (A) and Reason (R). Each of these questions also has four alternative choices, any one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) A is true, R is true; R is a correct explanation for A.

(b) A is true, R is true; R is not a correct explanation for A.

(c) A is true; R is false.

(d) A is false; R is true.

28. Assertion (A) If $\frac{3}{5}$ th of a cistern is filled in 1 min, the time needed to fill the rest is 40 min.

Reason (R) If a pipe can fill/empty $\frac{1}{m}$ part of a tank in 1 h, then it can fill the whole tank in m h.

29. Assertion (A) A cistern can be filled with water by a pipe in 5 h and it can be emptied by a second pipe in 4 h.

If both the pipes are opened when the cistern is full. The time in which it will be emptied the cistern is 20 h.

Reason (R) An inlet pipe A can fill a tank in n_A h and an outlet pipe B can empty the full tank B in n_B h ($n_B < n_A$). If on opening both the pipes together, the tank can be emptied in n_{AB} h, then $\frac{1}{n_{AB}} = \frac{1}{n_B} - \frac{1}{n_A}$.

30. Assertion (A) A boy and girl together fill a cistern with water. The boy pours 4 L of water in every 3 min and the girl pours 3 L in every 4 min. The time taken by both to fill 100 L of water in the cistern is 36 min.

Reason (R) If a pipe can fill a tank in x h and another pipe can empty the full tank in y h (where

$y > x$), then on opening both pipes, the part filled in 1 h = $\frac{1}{x} - \frac{1}{y}$.

31. Assertion (A) A tap can fill a tank in 48 min whereas another tap can empty it in 2 h. If both the taps are opened at 9 : 40 AM, the tank will be filled at 11 AM.

Reason (R) If a pipe fills the tank in m h and another pipe empty the tank in n h ($m < n$), then time taken to fill the tank is $\frac{mn}{n - m}$.

32. Assertion (A) Two pipes A and B can fill a tank in 15 min and 20 min respectively. Both the pipes are opened together but after 4 min, pipe A is turned off. The total time required to fill the tank is 15 min.

Reason (R) If a tank of capacity (volume) V units is filled by a pipe in n h, then part of the tank filled in 1 h $\frac{V}{n}$ units.

33. Assertion (A) Two pipes A and B can fill a tank in 20 h and 30 h respectively. If both the pipes are opened simultaneously, then pipe A should be closed after 8 h. So that the tank is filled in 18 h.

Reason (R) Time taken to fill the tank is taken positive (+ ve) and time taken to empty a tank is taken negative.

34. Assertion (A) Pipe A can fill the tank in

8 h and pipe B can fill it in 12 h. If pipe A is opened at 7:00 am and pipe B is opened at 9:00 am, then the tank will be full at 12:48 am.

Reason (R) Two pipe A and B separately can fill a tank in n_A and n_B h respectively. If the tank can be filled completely when pipe A fills for 'a' h and pipe B for 'b' h, then $\frac{a}{n_A} + \frac{b}{n_B} = 1$.

ANSWERS

28. (a) 29. (a) 30. (d) 31. (a) 32. (d) 33. (b) 34. (d)

(SHORT/LONG ANSWER QUESTIONS)

35. Two pipes A and B can fill a tank in 24 hours and 30 hours respectively. If the pipes are opened together in the empty tank, how much time will be taken by them to fill it?

36. Two pipes A and B can fill a tank in 18 hours and 6 hours respectively. If both the pipes are opened simultaneously, how much time will be taken to fill the tank?

37. Pipe A can fill a tank in 30 minutes and pipe B can fill it in 45 minutes. If both the pipes are opened in the empty tank, how much time will they take to fill it?

38. Two pipes A and B together can fill a cistern in 4 hours. Had they been opened separately, then B would have taken 6 hours more than A to fill the cistern. How much time will be taken by pipe A alone to fill the cistern?

39. Two pipes A and B can fill a tank in 15 minutes and 20 minutes respectively. Both the pipes are opened together but after 4 minutes, pipe A is turned off. What is the total time required to fill the tank?

40. Two pipes A and B can separately fill a cistern in $37\frac{1}{2}$ minutes and 45 minutes respectively. Both the pipes are opened. After how much time pipe B should be turned off so that the cistern is filled in just half an hour?

41. A cistern has two pipes. One can fill it with water in 8 hours and other can empty in 5 hours. In how many hours will the cistern be emptied if both the pipes are opened together when $\frac{3}{4}$ of the cistern is already full of water?

42. A tap can fill a tank in 48 minutes whereas another tap can empty it in 2 hours. If both the taps are opened at 9 : 40 AM, at what time the tank will be filled?

43. Two pipes can fill a tank in 12 hours and 16 hours respectively. A third pipe can empty the tank in 30 hours. If all the three pipes are opened and function simultaneously, how much time

will the tank take to be full?

44. A tap can completely fill a water tank in 8 hours. The water tank has a hole in it through which the water leaks out. The leakage will cause the full water tank to get empty in 12 hours. How much time will it take for the tap to fill the tank completely with the hole?

45. Three pipes A, B and C are connected to a tank. Out of the three A and B are inlet pipes and C is the outlet pipe. If opened separately, A fills the tank in 10 hours, B fills the tank in 12 hours and C empties the tank in 30 hours. If all three are opened simultaneously, how much time does it take to fill the tank?

46. Three pipes A, B and C are connected to a tank. Out of three, A is the inlet pipe and B and C are outlet pipes. If opened separately, A fills the tank in 10 hours, B empties the tank in 12 hours and C empties the tank in 30 hours. If all three pipes are opened simultaneously, how much time does it take to fill the empty tank?

47. Three pipes A, B and C are connected to a cistern. Out of three A and B are the inlet pipes and C is the outlet pipe. If opened separately, A fills the tank in 10 hours and B fills the tank in 30 hours. If all the three pipes are opened together, it takes 30 minutes extra than if only A and B are opened. How much time does it take to empty the tank if only pipe C is opened?

48. Time required by two pipes A and B working separately to fill a tank is 36 minutes and 45 minutes respectively. Another pipe C can empty the tank in 30 minutes. Initially, A and B are opened and after 7 minutes, C is also opened. In how much more time the tank would be completely filled?

49. Two pipes A and B can fill a tank in 20 hours and 30 hours respectively. If both the pipes are opened simultaneously, find after how much time should pipe B be closed so that the tank is full in 18 hours?

50. Two pipes A and B can fill a tank in 15 hours and 20 hours respectively while a third pipe C can empty the full tank in 25 hours. All the three pipes are opened in the beginning. After 10 hours, pipe C is closed. In how many hours will the tank be full?

51. Two pipes A and B can fill a tank in 15 and 12 hours respectively and a third pipe C can empty it in 4 hours. If the pipes A, B and C are opened at 8 AM, 9 AM and 11 AM respectively, at what time will the tank be fully empty?

52. Two pipes A and B can fill a tank separately in 20 hours and 30 hours respectively. Both the pipes are opened to fill the tank but when the tank is one-third full, a leak develops in the tank through which one third water supplied by both the pipes goes out. How long will it take to fill the tank?

53. A leak in the bottom of a tank can empty the full tank in 8 hours. An inlet pipe fills water at the rate of 6 litres a minute. When the tank is full, the inlet is opened but due to the leak, the tank is empty in 12 hours. How many litres does the tank hold?

54. Two pipes A and B can fill a cistern in 14 hours and 16 hours respectively. Pipes A and B are opened simultaneously and it is found that due to leakage in the bottom, 32 minutes extra are taken for the cistern to be filled up. If the cistern is full, in what time would the leak empty it?

55. Two inlet pipes A and B can fill a tank in 60 minutes and 30 minutes respectively. An outlet pipe C drains 24 litres of water in a minute. If all three pipes are opened when the tank is full, the tank gets emptied in 60 minutes. How much water can the tank hold?

56. Two pipes A and B can fill a cistern in 32 minutes and 48 minutes respectively. Both the pipes being opened, find when pipe B be closed so that the cistern gets filled in 24 minutes?

ANSWERS

35. 13 hours, 20 minutes 36. $4\frac{1}{2}$ hours 37. 18 minutes 38. 6 hours 39. 14 minutes, 40 seconds
40. 9 minutes 41. 10 hours 42. 11 AM 43. $8\frac{8}{9}$ hours 44. 24 hours 45. 6 hours 40 minutes 46. 60

hours 47. 120 hours 48. 39 seconds 49. 3 hours 50. 12 hours 51. 2 : 40 PM 52. 16 hours 53. 8640 litres 54. 112 hours 55. 360 litres 56. 12 minutes

RACES AND GAMES

- In a 100 m race, Ajay runs at the speed of 4 km/h. Ajay gives Brijesh a start of 4 m and still beats him by 15 s. The speed of Brijesh is
(a) 3.89 km/h (b) 3.54 km/h (c) 3 km/h (d) 3.29 km/h
- In a 100 m race, A runs at a speed of $\frac{5}{6}$ m/s. If A gives a start of 4 m to B and still beats him by 12 s. Then, the speed of B is
(a) $\frac{5}{4}$ m/s (b) $\frac{7}{5}$ m/s (c) $\frac{4}{3}$ m/s (d) $\frac{6}{5}$ m/s
- In a 400 m race, A runs at a speed of 16 m/s. If A gives B a start of 16 m and still beats him by 40 s. Then, what will be the speed of P?
(a) 6 m/s (b) 8 m/s (c) 15 m/s (d) 5.9 m/s
- In a 200 m race, A runs at a speed of 2 m/s. If A gives B a start of 10 m and still beats him by 5 s. Then, what will be the speed of B?
(a) $1\frac{17}{21}$ m/s (b) $3\frac{17}{21}$ m/s (c) 21 m/s (d) 2 m/s
- In 1 km race, A beats B by 36 m or 18 s. A's time over the course is
(a) 400 s (b) 482 s (c) 492 s (d) 488 s
- In a 1000 m race, X beats Y by 140 m or 14 s. The X's time over the course will be
(a) 86 s (b) 90 s (c) 95 s (d) 76 s
- In 1 km race, P beats Q by 72 m or 12 s. The P's time over the course is
(a) $155\frac{2}{3}$ s (b) 151s (c) $154\frac{2}{3}$ s (d) 160 s
- P covers 1 km in 4 min 40 s, while Q covers the same distance in 5 min. P defeat Q by a distance
(a) 66 m (b) $33\frac{1}{3}$ m (c) 60 m (d) $66\frac{2}{3}$ m
- X covers 1 km in 8 min 40 s while Y covers the same distance in 10 min. X defeat Y by a distance of
(a) $13\frac{1}{3}$ m (b) $133\frac{2}{3}$ m (c) $133\frac{2}{5}$ m (d) $133\frac{1}{3}$ m
- A can run 40 m while B runs 50 m. In 1 km race, B beats A by a distance of
(a) 175 m (b) 225 m (c) 335 m (d) 200 m
- Raman covers 1 km in 8 min while Suman covers the same distance in 10 min. Raman beat Suman by a distance of
(a) 150 m (b) 65 m (c) 190 m (d) 200 m
- Yogesh can run 1 km in 6 min 20 s and Vijay can cover the same distance in 6 min 40 s. Yogesh beat Vijay by a distance of
(a) 50 m (b) 90 m (c) 45 m (d) 30 m
- A can run 22.5 m while B runs 25 m. B beat A in a kilometer race by
(a) 110 m (b) 70 m (c) 90 m (d) 100 m
- A can run 1 km in 5 min and B can run the same distance in 6 min. If the race between A and B ends in a dead heat in a 1 km race, A has to give a start of
(a) 100 m (b) 120 m (c) $166\frac{2}{3}$ m (d) $133\frac{1}{3}$ m
- Arun and Bhaskar start from place P at 6:00 am and 7:30 am, respectively and run in the same direction. Arun and Bhaskar run at 8 km/h and 12 km/h, respectively. Bhaskar overtakes Arun at
(a) 10:30 am (b) 9:00 am (c) 11:30 am (d) 1:00 am
- A runs $1\frac{2}{3}$ times as fast as B. A gives B a start of 80 m. If A and B reach at the winning point at the same time, then the distance between the winning point and starting point is

- (a) 200 m (b) 300 m (c) 270 m (d) 160m
17. A runs $1\frac{3}{4}$ times as fast as B. A gives B a start of 30 m. If A and B reach at the winning point at the same time, then the distance between the winning point and starting point is
 (a) 52 m (b) 75 m (c) 69 m (d) 70 m
18. In 1 km race, the ratio of the speeds of two contestants L and M is 3 : 4. L has a start of 280 m. Then, L wins by how many metres?
 (a) 40 (b) 60 (c) 45 (d) 65
19. Two men A and B run a 500 m race. A having 140 m start their speeds are 3 : 4. Then, A wins by
 (a) 10 m (b) 20 m (c) 40 m (d) 60 m
20. A and B take part in 100 m race. A runs at 5 km/h. A gives B a start of 8 m and still beats him by 8 s. The speed of B is
 (a) 5.15 km/h (b) 4.14 km/h (c) 4.25 km/h (d) 4.4 km/h

Assertion-Reasoning MCQs

Directions (Q. Nos. 21-30) Each of these questions contains two statements Assertion (A) and Reason (R). Each of the questions has four alternative choices, any one of the which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

- (a) A is true, R is true; R is a correct explanation of A.
 (b) A is true, R is true; R is not a correct explanation of A.
 (c) A is true; R is false
 (d) A is false; R is true.
21. Assertion (A) A can run 22.5 m while B runs 25 m. Then, in a kilometer race, B beats A by 100 m.
 Reason (R) A beats B by x m means that in the same time, while A reached the winning point, whereas, B is behind A by V m.
22. Assertion (A) In a 50 m race, A can give a start of 5 m to B and a start of 14 m to C. Then, B gives a start of 40 m to C.
 Reason (R) A gives a start of x m means that before the start of race, A is at the starting point and B is ahead of A by V m.
23. Assertion (A) In a 800 m race, A beats B by 74 m and in 600 m race, B beats C by 50 m. Then, A beats C in a race of 500 m by 84.06 m.
 Reason (R) A beats B by x m means that in the same time, while B reached the winning point, whereas, A is behind B by 'x' m.
24. Assertion (A) In a kilometer race, A beats B by 30 s and B beats C by 15 s. If A beats C by 180 m, then A takes 205 s to run 1 km.
 Reason (R) A beats B by x m means that in the same time, while A reached the winning point, whereas, B is behind by 'x' m.
25. Assertion (A) In a game of 100 points, A can give B 20 points and C 28 points. Then, B can give C 8 points.
 Reason (R) In a game of 100 points, if A scores 100 points, while B scores 80 points, then we say that A can give B 20 points.
26. Assertion (A) At a game of billiards, A can give B 15 points in a game of 60 and A can give C 20 points in a game of 60. Then, B can give C 10 points in a game of 90.
 Reason (R) A game of 100 means that a person among the contestants, who scores 100 points first is the winner.
27. Assertion (A) A can run a kilometer in 4 min 50 s and B in 5 min. Then, A must give 10 s start to B in a kilometer race, so that the race may end in a dead heat.
 Reason (R) A can give B a start of t min means that A will start 't' min after B starts from the starting point and both A and B reach the finishing point at the same time.

28. Assertion (A) In a 100 m race, A runs with 6 km/h. If A gives B a start of 4 m and still beats him by 12 s. Then, speed of B is 4.8 km/h.

Reason (R) A beats B by x m means that in the same time, while A reached the winning point, whereas, B is behind A by 'x' m.

29. Assertion (A) A and B run 1 km and A wins B by 1 min. A and C also run 1 km and A wins by 375 m. B and C run 1 km and B wins by 30 s. Then, C covers 1 km in 240 s.

Reason (R) If A gives B x m and t min, then B covers x m in extra time of t min.

So, speed of B is $\frac{x}{t}$ m/s.

30. Assertion (A) A runs 4 times as fast as B. If A gives B a start of 60 m. Then, the goal is 180 m away on the race course, so that A and B reach it at the same time.

Reason (R) A gives B a start of x m means that before the start of race, A is at the starting points and B is ahead of A by 'x' m.

ANSWERS

21. (a) 22. (d) 23. (c) 24. (b) 25. (a) 26. (b) 27. (a) 28. (b) 29. (c) 30. (d)

(SHORT/LONG ANSWER QUESTIONS)

31. In a kilometer race, A, B and C are three participants. A can give B a start of 50 meters and C a start of 69 meters. In the same race, how much start can B give to C?

32. A can run a kilometer in 4 minutes 50 seconds and B in 5 minutes. How many meters start can A give B in a kilometer race, so that the race may end in a dead heat.

33. In a 100 meter race, A runs with 6 km/hr. If a gives B a start of 4 meters and still beats him by 12 seconds. What is the speed of B?

34. If A can run 1 kilometer in 190 seconds and B can run the same distance in 200 seconds, then by what distance A can beat B?

35. A and B run 1 kilometer and A wins B by 1 minute. A and C also run 1 kilometer and A wins by 375 meters. B and C run 1 kilometer and B wins by 30 seconds. Find the time taken by each to run one kilometer distance.

36. In a 100 m race, A runs at 1.66 m/sec. If A gives B a start of 4 m and still beats him by 12 seconds, what is B's speed?

37. In a race of 200 m, A can beat B by 31 m and C by 18 m. In a race of 350 m by how much distance will C beat B?

38. In a racing over a distance 'd' at a uniform speed, A can beat B by 20 meters, B can beat C by 10 meters and A can beat C by 28 meters. Find distance d.

39. A runs $\frac{5}{3}$ times as fast as B. If A gives B a start of 80 m, how far must the winning post from the starting point be so that A and B reach at the same time?

40. In a game of billiards, A can give B 20 points in 60 and A can give C 30 points in 60. How many points can B give C in a game of 100?

ANSWERS

31. 20 meters 32. $\frac{100}{3}$ meters 33. 4.8km/hr 34. 50 meters 35. 150 seconds, 210 seconds, 240 seconds 36. 1.33 m/sec 37. 25 m 38. 100 meters 39. 200 meters 40. 25 points

NUMERICAL INEQUALITIES

1. Ravi goes to market with Rs. 200 to buy rice, which is available in packets of 1 kg. The price of one packet of rice is Rs. 30. If x denotes the number of packets of rice which he buys, then the total amount spent by him is Rs. 30x. The mathematical formulation of given problem is (a) $30x > 200$ (b) $30x < 200$ (c) $30x \leq 200$ (d) $30x \geq 200$

2. Reshma has Rs. 120 and wants to buy some registers and pens. The cost of one register is Rs. 40 and that of a pen is Rs. 20. The mathematical formulation of given problem is

(a) $40x + 20y = 120$ (b) $40x + 20y > 120$ (c) $40x + 20y \leq 120$ (d) $40x + 20y \geq 120$

3. Two real numbers or two algebraic expressions related by the symbol '<', '>', '≤' or '≥' forms an ...X... Here, X refers to

(a) equation (b) inequality (c) set (d) None of these

4. Which of the following is/are double inequalities?

(a) $3 < 5 < 7$ (b) $3 \leq x < 5$ (c) $2 < y \leq 4$ (d) All of these

5. If $(-8)(-2) > (-7)(-2)$, then

(a) $-a > -7$ (b) $-8 < -7$ (c) $-8 \geq -7$ (d) $-8 \leq -7$

6. The solution set of the inequality $4x + 3 < 6x + 7$ is

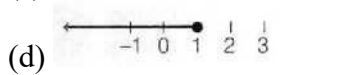
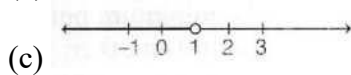
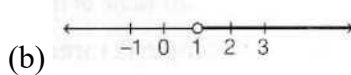
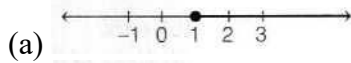
(a) $[-2, \infty)$ (b) $(-\infty, -2)$ (c) $(-2, \infty)$ (d) None of these

7. The set of real x satisfying the

inequality $\frac{5-2x}{3} \leq \frac{x}{6} - 5$ is

(a) $(-\infty, 8)$ (b) $(8, \infty)$ (c) $[8, \infty)$ (d) $(-\infty, 8]$

8. The graph of the solutions of inequality $\frac{3x-4}{2} \geq \frac{x+1}{4} - 1$ on number line is



9. The marks obtained by a student of class XI in first and second terminal examinations are 62 and 48, respectively. The minimum marks he should get in the annual examination to have an average of at least 60 marks, are

(a) 70 (b) 50 (c) 74 (d) 48

10. If $3 \leq 3t - 18 \leq 18$, then which one of the following is true?

(a) $15 \leq 2t + 1 \leq 20$ (b) $8 \leq t < 12$ (c) $8 \leq t + 1 \leq 13$ (d) $21 \leq 3t \leq 24$

11. If $-3 \leq 4 - \frac{7x}{2} \leq 18$, then

(a) $x \in (-\infty, -4]$ (b) $x \in [-4, \infty)$ (c) $x \in [-4, 2]$ (d) $x \in [2, \infty)$

12. If $|x| < 3$ and x is a real number, then

(a) $x \geq 3$ (b) $-3 < x < 3$ (c) $x \leq -3$ (d) $-3 \leq x \leq 3$

13. x and b are real numbers. If $b > 0$ and $|x| > b$, then

(a) $x \in (-b, \infty)$ (b) $x \in [-\infty, b)$ (c) $x \in (-b, b)$ (d) $x \in (-\infty, -b) \cup (b, \infty)$

14. If $p > 0$ and $q < 0$, then $p + q \dots A \dots p$. Here, A refers to

(a) $>$ (b) $<$ (c) \geq (d) \leq

15. The cost and revenue function of a product are given by $C(x) = 20x + 4000$ and $R(x) = 60x + 2000$ respectively, where x is the number of items produced and sold. Then, the number of items must be sold to realise some profit is given by

(a) $x=50$ (b) $x>50$ (c) $x<50$ (d) None of these

16. The length of a rectangle is three times the breadth. If the minimum perimeter of the rectangle is 160 cm, then

(a) breadth > 20 cm (b) length < 20 cm (c) breadth ≥ 20 cm (d) length ≤ 20 cm

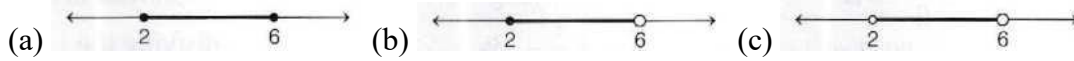
17. The minimum value of $f(x) = |3 - x| + 7$ is

(a) 0 (b) 6 (c) 7 (d) 8

18. The number of pairs of consecutive even positive integers, both of which are larger than 8, such that their sum is less than 25, is

(a) 1 (b) 2 (c) 3 (d) 4

19. The solutions of the system of inequalities $3x - 7 < 5 + x$ and $11 - 5x \leq 1$ on the number line is



(d) None of the above

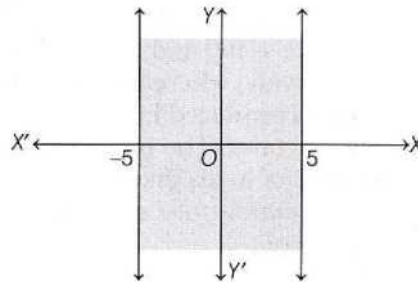
20. A solution is to be kept between 68°F and 77°F . The range in temperature in degree Celsius ($^{\circ}\text{C}$), if the Celsius/Fahrenheit (F) conversion formula is given by $F = \frac{9}{5}C + 32$, is

- (a) between 20°C and 22°C (b) between 20°C and 25°C (c) between 36°C and 45°C
 (d) between 4°C and 5°C

21. The set of admissible values of x such that $\frac{2x+3}{2x-9} < 0$, is

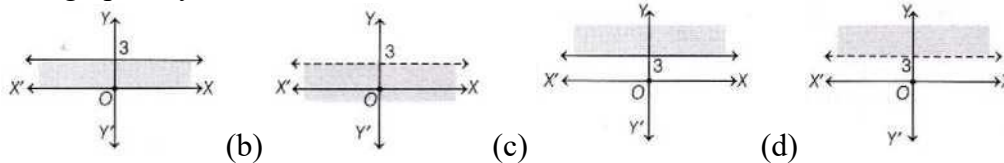
- (a) $(-\infty, \frac{-9}{2}) \cup (\frac{9}{2}, \infty)$ (b) $(\frac{-3}{2}, 0)$ (c) $(0, \frac{9}{2})$ (d) $(\frac{-3}{2}, \frac{9}{2})$

22. The inequality representing the following graph is

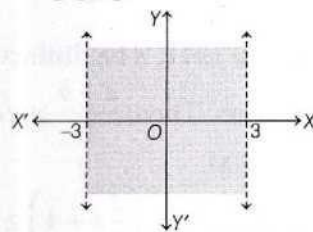


- (a) $|x| < 5$ (b) $|x| \leq 5$ (c) $|x| > 5$ (d) $|x| \geq 5$

23. The graph of $y < 3$ is

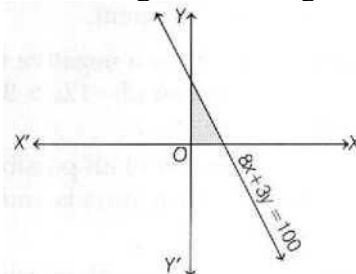


24. The inequality representing the following graph is



- (a) $|x| < 3$ (b) $|x| \leq 3$ (c) $|x| > 3$ (d) $|x| \geq 3$

25. Which of the following represent the region in the figure?



- (a) $8x + 3y \geq 100, x \geq 0, y \geq 0$ (b) $8x + 3y \leq 100, x \geq 0, y \geq 0$ (c) $8x + 3y \geq 100$ (d) $8x + 3y \leq 100$

26. If $3x + 8 > 2$, then which of the following is true?

- (a) $x \in \{-1, 0, 1, 2, \dots\}$, when x is an integer (b) $x \in [-2, \infty)$, when x is a real number
 (c) Both (a) and (b) (d) None of the above

ANSWERS

1. (b) 2. (c) 3. (b) 4. (d) 5. (b) 6. (c) 7. (c) 8. (a) 9. (a) 10. (c)
 11. (c) 12. (b) 13. (d) 14. (b) 15. (b) 16. (c) 17. (c) 18. (a)
 19. (b) 20. (b) 21. (d) 22. (b) 23. (b) 24. (a) 25. (b) 26. (a)

Assertion-Reasoning MCQs

Directions (Q. Nos. 27-33) Each of these questions contains two statements Assertion (A) and Reason (R). Each of the questions has four alternative choices, any one of the which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) A is true, R is true; R is a correct explanation of A.

(b) A is true, R is true; R is not a correct explanation of A.

(c) A is true; R is false

(d) A is false; R is true.

27. Assertion (A) If a is any positive real number, then $a + \frac{1}{a} \geq 2$.

Reason (R) Let a, b be distinct positive real numbers. Then, $\frac{a+b}{2} > \sqrt{ab}$ i.e. $AM > GM$.

28. Assertion (A) If $\frac{1}{2} \left(\frac{3}{5}x + 4 \right) \frac{1}{3} (x - 6)$,

$x \in \mathbb{R}$, then $x \in [120, \infty]$.

Reason (R) A solution of an inequation is the value(s) of the variable (s) that makes it a true statement.

29. Assertion (A) If x is a negative integer, then the solution set of $-12x > 30$ is $\{\dots, -5, -4, -3\}$.

Reason (R) The set of all possible solutions of an inequation is known as its solution set.

30. Assertion (A) The length of rectangle is double the breadth. If the minimum perimeter of the rectangle is 120 cm. then breadth ≥ 20 cm.

Reason (R) Perimeter of rectangle

= Length + Breadth.

31. Assertion (A) The solution set of system of linear inequalities.

$3x - 1 \geq 5, 2x - 3 > 7, x \in \mathbb{R}$ is $(5, \infty)$.

Reason (R) The solution set of a system of linear inequations in one variable is the intersection of the solution sets of the linear inequations.

32. Assertion (A) If $|x - 5| < 2, x \in \mathbb{R}$, then $x \in [3, 7]$.

Reason (R) $|x| < 1$ iff $-1 < x < 1$.

33. Assertion (A) If $2x + 1 < |2x + 1|, x \in \mathbb{R}$, then $x \in \left(-\infty, \frac{1}{2}\right)$

Reason (R) $|x| \leq |y|$ iff $x^2 \geq y^2$.

ANSWERS

27. (a) 28. (d) 29. (a) 30. (c) 31. (a) 32. (d) 33. (b)

(SHORT/LONG ANSWER QUESTIONS)

Solve the following Linear inequalities

34. $2(3 - x) \geq \frac{x}{5} + 4$ 35. $\frac{3x-2}{5} \leq \frac{4x-3}{2}$ 36. $-(x - 3) + 4 < 5 - 2x$

37. $\frac{x}{5} < \frac{3x-2}{4} - \frac{5x-3}{5}$ 38. $\frac{2(x-1)}{5} \leq \frac{3(2+x)}{7}$ 39. $\frac{5x}{2} + \frac{3x}{4} \geq \frac{39}{4}$

40. $\frac{x-1}{3} + 4 < \frac{x-5}{5} - 2$ 41. $\frac{2x+3}{4} - 3 < \frac{x-4}{3} - 2$ 42. $\frac{5-2x}{3} < \frac{x}{6} - 5$

43. $\frac{4+2x}{3} \geq \frac{x}{2} - 3$ 44. $\frac{2x+3}{5} - 2 < \frac{3(x-2)}{5}$ 45. $x - 2 \leq \frac{5x+8}{3}$

46. $\frac{6x-5}{4x+1} < 0$ 47. $\frac{2x-3}{3x-7} > 0$ 48. $\frac{3}{x-2} < 1$

49. $\frac{1}{x-1} \leq 2$ 50. $\frac{5x+8}{4-x} < 2$ 51. $\frac{x}{x-5} > \frac{1}{2}$

52. Find all pairs of consecutive odd positive integers, both of which are smaller than 10, such that their sum is more than 11.

53. Find all pairs of consecutive odd natural number, both of which are larger than 10, such that their sum is less than 40.

54. Find all pairs of consecutive even positive integers, both of which are larger than 5, such that their sum is less than 23.

55. The marks scored by Rohit in two tests were 65 and 70. Find the minimum marks he should score in the third test to have an average of at least 65 marks.

56. A solution is to be kept between 86° and 95° F. What is the range of temperature in degree Celsius, if the Celsius (C)/Fahrenheit (F) conversion formula is given by $F = \frac{9}{5}C + 32$.

57. A solution is to be kept between 30° C and 35° C What is the range of temperature in degree Fahrenheit?

58. To receive grade 'A' in a course, one must obtain an average of 90 marks or more in five papers each of 100 marks. If Shikha scored 87,95,92 and 94 marks in first four papers, find the minimum marks that she must score in the last paper to get grade 'A' in the course.

59. A company manufactures cassettes and its cost and revenue functions for a week are $C = 300 + \frac{3}{2}x$ and $R = 2x$ respectively, where x is the number of cassettes produced and sold in a week. How many cassettes must be sold for the company to realize a profit?

60. The longest side of a triangle is three times the shortest side and the third side is 2 cm shorter than the longest side if the perimeter of the triangles at least 61 cm, Find the minimum length of the shortest-side.

61. How many litres of water will have to be added to 1125 litres of the 45% solution of acid so that the resulting mixture will contain more than 25% but less than 30% acid content?

62.. A solution of 8% boric acid is to be diluted by adding a 2% boric acid solution to it. The resulting mixture is to be more than 4% but less than 6% boric acid. If there are 640 litres of the 8% solution, how many litres of 2% solution will have to be added?

63. The water acidity in a pool is considered normal when the average pH reading of three daily measurements is between 7.2 and 7.8. If the first two pH reading are 7.48 and 7.85, find the range of pH value for the third reading that will result in the acidity level being normal.

ANSWERS

34. $(-\infty, 10/11]$

35. $[11/14, \infty)$

36. $(-\infty, -2)$

37. $(-\infty, 2/9)$

38. $[-44, \infty)$

39. $[3, \infty)$

40. $(-\infty, -50)$

41. $(-\infty, -13/2)$

42. $(8, \infty)$

43. $[-26, \infty)$

44. $(-1, \infty)$

45. $[-7, \infty)$

46. $(-1/4, 5/6)$

47. $(-\infty, 3/2) \cup (7/3, \infty)$

48. $(-\infty, 2) \cup (5, \infty)$

49. $(-\infty, 1) \cup [3/2, \infty)$

50. $(-\infty, 0) \cup (4, \infty)$

51. $(-\infty, -5) \cup (5, \infty)$

52. $(5, 7), (7, 9)$

53. $(11, 13), (13, 15), (15, 17), (17, 19)$

54. $(6, 8), (8, 10), (10, 12)$

55. 60

56. Between 30° C and 35° C

57. Between 86° F and 95° F

58. 82 marks

59. More than 600

60. 9 cm

61. More than 562.5 litres but less than 900 litres

62. More than 320 litres but less than 1280 litres 63. Between 6.27 and 8.07

(CASE STUDY/SOURCE BASED QUESTIONS)

63. A company produces certain items. The manager in the company used to make a data record on daily basis about the cost and revenue of these items separately. The cost and revenue

functions of a product are given by

$$C(x) = 20x + 4000 \text{ and}$$

$$R(x) = 60x + 2000, \text{ respectively, where } x \text{ is the number of items produced and sold.}$$

Based on the above information, answer the following questions.

(i) How many items must be sold to realize some profit.

(ii) Also, if the cost and revenue functions of a product are given by

$$C(x) = 2x + 400 \text{ and } R(x) = 6x + 20$$

respectively, where x is the number of items produced by the manufacturer. The minimum number of items that the manufacturer must sell to realize some profit, is _____

64. In drilling world's deepest hole, the Kola superdeep Borehole, the deepest man made hole on Earth and deepest artificial point on Earth, as a result of a scientific drilling project, it was found that the temperature T in degree, x km below the surface of Earth, was given by

$$T = 30 + 25(x - 3), 3 < x < 15$$

If the required temperature lies between 200°C and 300°C , then

Based on the above information, answer the following questions.

(i) The depth, x will lie between _____

(ii) Solve for x

$$-13x + 2 > 18 \text{ or } 13x + 15 \leq -4$$

ANSWERS

$$63. (i) x > 50 \text{ (ii) } 96 \text{ 64. (i) } 9 \text{ km and } 13 \text{ km (ii) } x \leq \frac{-19}{13}$$

MATRICES AND DETERMINANTS

- A matrix having m rows and n columns with $m \neq n$ is said to be a
 - rectangular matrix
 - square matrix
 - identity matrix
 - scalar matrix
- The matrix $P = \begin{bmatrix} 0 & 0 & 4 \\ 0 & 4 & 0 \\ 4 & 0 & 0 \end{bmatrix}$ is a
 - square matrix
 - diagonal matrix
 - unit matrix
 - None of these
- The order of matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & -1 \end{bmatrix}$ is
 - 3×2
 - 2×3
 - 2×2
 - 3×3
- If $A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 3 & 4 \end{bmatrix}$, then order of the matrix A is 3×2 .
 - True
 - False
 - Can't say
 - Partially True/False
- If A is a matrix having 3 rows and 2 columns, then number of elements in matrix A is
 - 5
 - 6
 - 18
 - 1
- If a matrix has 18 elements, then the number of its possible order is
 - 2
 - 4
 - 6
 - 8

7. Suppose 3×3 matrix $A = [a_{ij}]$, whose elements are given by $a_{ij} = i^2 - j^2$, then a_{32} is equal to
 (a) 5 (b) 1 (c) 2 (d) 3
8. If $\begin{bmatrix} 2x + y & 4x \\ 5x - 7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y - 13 \\ y & x + 6 \end{bmatrix}$, then
 (a) $x = 3, y = 1$ (b) $x = 2, y = 3$
 (c) $x = 2, y = 4$ (d) $x = 3, y = 3$
9. Two matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ are said to be equal, if they are of same order and for all i and j , their elements are
 (a) $a_{ij} = b_{ij}$ (b) $a_{ij} = b_{ji}$ (c) $a_{ij} = -b_{ij}$ (d) $a_{ij} + b_{ij} = 0$
10. If $\begin{bmatrix} 1 + x & 2 \\ y - 1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix}$, then $x = -1$ and $y = 2$.
 (a) True (b) False
 (c) Can't say (d) Partially True/False
11. If $x \begin{bmatrix} 2 \\ 3 \end{bmatrix} + y \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$, then the value of x and y are
 (a) -4, 3 (b) 3, -4 (c) 3, -3 (d) 4, -4
12. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 & 2 \\ 4 & 3 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $D = \begin{bmatrix} 4 & 6 & 8 \\ 5 & 7 & 9 \end{bmatrix}$, then which of the following is defined?
 (a) $A + B$ (b) $B + C$ (c) $C + D$ (d) $B + D$
13. If $\begin{bmatrix} 1 & 2 \\ -2 & -b \end{bmatrix} + \begin{bmatrix} a & 4 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 0 \end{bmatrix}$, then $a^2 + b^2$ is equal to
 (a) 20 (b) 22 (c) 12 (d) 10
14. Given $A = \begin{bmatrix} \sqrt{3} & 1 & -1 \\ 2 & 3 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & \sqrt{5} & 1 \\ -2 & 3 & 1/2 \end{bmatrix}$, then find $A + B$.
 (a) $\begin{bmatrix} 2 + \sqrt{3} & 1 + \sqrt{5} & 0 \\ 0 & 6 & 1/2 \end{bmatrix}$ (b) $\begin{bmatrix} \sqrt{3} + \sqrt{2} & 1 + \sqrt{2} & 3 \\ 0 & 6 & 0 \end{bmatrix}$
 (c) $\begin{bmatrix} \sqrt{3} + \sqrt{2} & 1 + \sqrt{3} & 4 \\ 0 & 1 & 1/2 \end{bmatrix}$ (d) $\begin{bmatrix} \sqrt{3} + 2 & 1 & 0 \\ 0 & 6 & 1 \end{bmatrix}$
15. If $2 \begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$, then find $(x - y)$.
 (a) 10 (b) 12 (c) 7 (d) 15
16. If $A = \begin{bmatrix} 4 & 4 \\ 2 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 4 \\ 3 & -5 \end{bmatrix}$, then find $A - B - C$.
 (a) $\begin{bmatrix} 2 & -2 \\ 0 & 9 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 3 \\ 6 & -4 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 4 \\ 6 & -5 \end{bmatrix}$ (d) $\begin{bmatrix} 4 & -3 \\ 5 & 0 \end{bmatrix}$
17. If $A = \begin{bmatrix} 1 & 3 \\ 5 & 2 \end{bmatrix}$, then $2A = \dots\dots\dots$

- (a) $\begin{bmatrix} 2 & 6 \\ 10 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 6 \\ 10 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & 3 \\ 5 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 3 \\ 5 & 4 \end{bmatrix}$
18. If $A = \begin{bmatrix} 3 & 1 & 1.5 \\ \sqrt{5} & 7 & -3 \\ 2 & 0 & 5 \end{bmatrix}$, then $3A$ is
- (a) $\begin{bmatrix} 9 & 3 & 4.5 \\ \sqrt{5} & 21 & -9 \\ 2 & 0 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} 9 & 3 & 4.5 \\ 3\sqrt{5} & 21 & -9 \\ 6 & 0 & 15 \end{bmatrix}$
- (c) $\begin{bmatrix} 9 & 3 & 1.5 \\ 3\sqrt{5} & 21 & -3 \\ 6 & 0 & 5 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 1 & 4.5 \\ \sqrt{5} & 21 & -6 \\ 2 & 0 & 15 \end{bmatrix}$
19. $x \begin{bmatrix} 2 \\ 3 \end{bmatrix} + y \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$, then the value of x is
- (a) 0 (b) 3 (c) 7 (d) 10
20. The value of x and y are respectively, if $\begin{bmatrix} x & y \\ 3y & x \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$.
- (a) 1, 1 (b) 2, 1 (c) 4, 3 (d) 3, 2
21. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$, then A^2 is equal to
- (a) 0 (b) $-A$ (c) 1 (d) $2A$
23. If $\begin{bmatrix} x & -5 & -1 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$, then the value of x is
- (a) $5\sqrt{5}$ (b) $\pm 4\sqrt{3}$ (c) $\pm 3\sqrt{5}$ (d) $\pm 6\sqrt{5}$
24. Let $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix}$. If $AB = BA$, then what is the value of x ?
- (a) -1 (b) 0
(c) 1 (d) Any real number
25. If $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then the value of k , so that $A^2 = 8A + kI$ is
- (a) 4 (b) 5 (c) 6 (d) -7
26. If $A = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$, then
- (a) AB is not defined (b) BA is not defined
(c) $AB = BA$ (d) $AB \neq BA$
27. Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}$ and $f(x) = x^2 + x - 1$, then find $f(A)$.
- (a) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 3 \\ 3 & 6 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 0 \\ 4 & 5 \end{bmatrix}$ (d) $\begin{bmatrix} 4 & 1 \\ 5 & 6 \end{bmatrix}$

28. Consider the matrices

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 3 & -2 & 1 \\ -1 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & -2 \\ 2 & 1 \\ 4 & 3 \end{bmatrix}, C = [1 \ 2 \ 6]$$

Then, which of the following is not defined?

- (a) CA (b) BA (c) AB (d) CB

29. If A is a square matrix such that $A^2 = I$, then the simplified value of

$$(A - I)^3 + (A + I)^3 - 7A$$

- (a) I (b) A^2 (c) A (d) A^{-1}

30. If $X = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix}$ and $A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$ satisfy the equation $AX = B$, then the matrix A is equal to

- (a) $\begin{bmatrix} -7 & 26 \\ 1 & -5 \end{bmatrix}$ (b) $\begin{bmatrix} 7 & 26 \\ 4 & 17 \end{bmatrix}$ (c) $\begin{bmatrix} -7 & -4 \\ 26 & 13 \end{bmatrix}$ (d) $\begin{bmatrix} -7 & 26 \\ -6 & 23 \end{bmatrix}$

31. If $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, then A^2 is equal to

- (a) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

32. The product $\begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ is equal to

- (a) $\begin{bmatrix} a^2 + b^2 & 0 \\ 0 & a^2 + b^2 \end{bmatrix}$ (b) $\begin{bmatrix} (a+b)^2 & 0 \\ (a+b)^2 & 0 \end{bmatrix}$
 (c) $\begin{bmatrix} a^2 + b^2 & 0 \\ a^2 + b^2 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

33. If $A = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$, then $A^2 + 2A - 3I$ is equal to

- (a) $\begin{bmatrix} 4 & -6 \\ 6 & 4 \end{bmatrix}$ (b) 0 (c) $\begin{bmatrix} -6 & 2 \\ -2 & 6 \end{bmatrix}$ (d) 5I

34. If matrix $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $A^2 = kA$, then the value of k is

- (a) 1 (b) 2 (c) 3 (d) 5

35. If $A = [a_{ij}]_{m \times n}$, then A' is equal to

- (a) $[a_{ji}]_{n \times m}$ (b) $[a_{ij}]_{m \times n}$ (c) $[a_{ji}]_{m \times n}$ (d) $[a_{ij}]_{n \times m}$

36. The transpose of the matrix $\begin{bmatrix} 4 & 3 & 1 \\ 1 & -2 & 3 \\ 4 & 5 & -1 \end{bmatrix}$ is

- (a) $\begin{bmatrix} 4 & 1 & 4 \\ 3 & -2 & 5 \\ 1 & 3 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 4 & 1 \\ 3 & 5 & -2 \\ 1 & -1 & 3 \end{bmatrix}$
 (c) $\begin{bmatrix} 4 & 3 & 1 \\ 1 & -2 & 3 \\ 4 & 5 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 4 & 3 & 1 \\ 4 & 5 & -1 \\ 1 & -2 & 3 \end{bmatrix}$

37. If $A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then $A^T - B^T$ is
- (a) $\begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -1 \end{bmatrix}$
(c) $\begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -2 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 4 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$
38. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$, then $A^T + B^T = \begin{bmatrix} 4 & 3 \\ 3 & 6 \end{bmatrix}$.
- (a) True (b) False
(c) Can't say (d) Partially True/False
39. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 1 \\ 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 6 & 4 \\ 7 & 3 \end{bmatrix}$, then $(A - B)'$ is equal to
- (a) $A' + B'$ (b) $A' - B'$ (c) $A'B'$ (d) $B'A'$
40. If $A = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}$ and $B = [1 \ 3 \ -6]$, then $(AB)'$ is equal to
- (a) $B'A'$ (b) $A'B'$ (c) AB' (d) BA'
41. For the matrix $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$, $A - A'$ is a
- (a) skew-symmetric matrix (b) symmetric matrix
(c) diagonal matrix (d) scalar matrix
42. The matrix $\begin{bmatrix} 0 & -5 & 8 \\ 5 & 0 & 12 \\ -8 & -12 & 0 \end{bmatrix}$ is a
- (a) diagonal matrix (b) symmetric matrix
(c) skew-symmetric matrix (d) scalar matrix
43. If A and B are symmetric matrices of same order, then $AB - BA$ is a
- (a) skew-symmetric matrix (b) symmetric matrix
(c) zero matrix (d) identity matrix
44. If $A = \begin{bmatrix} 4 & x + 2 \\ 2x - 3 & x + 1 \end{bmatrix}$ is symmetric, then what is x equal to?
- (a) 2 (b) 3 (c) -1 (d) 5
45. A square matrix A can be expressed as $A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$, where
- (a) $\frac{1}{2}(A + A')$ is a skew-symmetric matrix of A
(b) $\frac{1}{2}(A - A')$ is a symmetric matrix of A

- (c) $\frac{1}{2}(A - A')$ is a skew-symmetric matrix of A
 (d) None of the above
46. The value of the determinant $\begin{vmatrix} 1 & 6 \\ 0 & 9 \end{vmatrix}$ is
 (a) 9 (b) -9 (c) 6 (d) -6
47. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 6 \\ 1 & 1 \end{bmatrix}$, then $|A| + |B|$ is equal to
 (a) 0 (b) 1 (c) -3 (d) 3
48. If $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$, then x is
 (a) $\pm\sqrt{3}$ (b) $\pm\sqrt{2}$ (c) 1 (d) 0
49. The value of determinant $\begin{vmatrix} x & x+1 \\ x-1 & x \end{vmatrix}$ is equal to
 (a) 1 (b) 0 (c) x (d) x^2
50. If $A = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix}$, then $|AB|$ is equal to
 (a) 12 (b) 14 (c) 11 (d) -14
51. The value of the determinant
 $\Delta = \begin{vmatrix} 1 & 2 & 4 \\ -1 & 3 & 0 \\ 4 & 1 & 0 \end{vmatrix}$ is
 (a) -52 (b) 52 (c) 44 (d) -44
52. If there are two values of a, which produce determinants
 $\Delta = \begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86$, then the sum of these numbers is
 (a) 2 (b) 4 (c) -4 (d) 1
53. If $f(x) = \begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix}$, then
 (a) $f(a) = 0$ (b) $f(b) = 0$ (c) $f(0) = 0$ (d) $f(-1) = 0$
54. If $A = \begin{bmatrix} x & 1 \\ 2 & 4 \end{bmatrix}$ is singular matrix, then x is equal to
 (a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 1 (d) -1
55. For what value of k the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ is non-singular
 (a) $k \in \mathbb{R}$ (b) $k \in \mathbb{R} - \{6\}$ (c) $k \in \mathbb{Z}$ (d) $k \in \mathbb{Z}^+$
56. If A is a matrix of order 3 x 3, then $|3A|$ is equal to
 (a) $27|A|$ (b) $9|A|$ (c) $3|A|$ (d) $81|A|$

57. If A and B are matrices of order 3 and $|A| = 5$, $|B| = 3$, then
 $|3AB| = 27 \times 5 \times 3 = 405$.
 (a) True (b) False
 (c) Can't Say (d) Partially True/False
58. If $\Delta = \begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$, then Δ is equal to
 (a) 1 (b) -1 (c) 0 (d) 2
59. The area of the triangle, whose vertices are (3, 8), (-4, 2) and (5, 1), is
 (a) 60 sq units (b) 61 sq units (c) $\frac{61}{2}$ sq units (d) 30 sq units
60. If the points (2, -3), (λ , -1) and (0, 4) are collinear, then the value of λ is
 (a) $\frac{10}{7}$ (b) $\frac{7}{10}$ (c) $\frac{3}{10}$ (d) 1
61. The equation of line joining (2, 3) and (-1, 2) is
 (a) $x - 3y - 7 = 0$ (b) $x - 3y + 7 = 0$
 (c) $x + 3y - 7 = 0$ (d) $x + 3y + 7 = 0$
62. If the area of a ΔABD is 3 sq units with vertices A (1, 3), B (0, 0) and D (k, 0), then k is equal to
 (a) 3 (b) ± 3 (c) 2 (d) ± 2
63. The minors of the diagonal elements of the determinant $\begin{vmatrix} 3 & -1 & 2 \\ 4 & -1 & 3 \\ 2 & 0 & -1 \end{vmatrix}$ are
 (a) 1, 7, 1 (b) -1, 7, 1 (c) 1, -7, 1 (d) None of these
64. If $\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$, then the cofactor A_{21} is
 (a) $-(hc + fg)$ (b) $fg - hc$ (c) $fg + hc$ (d) $hc - fg$
65. If $M_{11} = -40$, $M_{12} = -10$ and $M_{13} = 35$ of the determinant $\Delta = \begin{vmatrix} 1 & 3 & -2 \\ 4 & -5 & 6 \\ 3 & 5 & 2 \end{vmatrix}$, then the value of Δ is
 (a) -80 (b) 60 (c) 70 (d) 100
66. The minor and cofactors of all the elements of the determinant $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$ is
 (a) $\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 4 & -2 \\ -3 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 4 & -2 \\ 1 & -3 \end{bmatrix}$
67. If A is a non-singular 3 x 3 matrix and B is its adjoint such that $|B| = 64$, then $|A|$ is
 (a) 64 (b) ± 64 (c) ± 8 (d) 18

68. If $A = \begin{bmatrix} 2 & 1 & 0 \\ 3 & 1 & 2 \\ 0 & 4 & -1 \end{bmatrix}$, then $|adj(A)|$ is equal to
 (a) 11 (b) 12 (c) 225 (d) -225
69. If $A = \begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix}$, then A^{-1} is equal to
 (a) $\begin{bmatrix} 1 & -\frac{1}{3} \\ 0 & \frac{1}{3} \end{bmatrix}$ (b) $\begin{bmatrix} 1 & \frac{1}{3} \\ 0 & -\frac{1}{3} \end{bmatrix}$ (c) $\begin{bmatrix} 0 & \frac{1}{3} \\ 1 & -\frac{1}{3} \end{bmatrix}$ (d) $\begin{bmatrix} 0 & -\frac{1}{3} \\ 1 & 1 \end{bmatrix}$
70. If $A = \begin{bmatrix} 1 & 4 \\ 3 & 15 \end{bmatrix}$, then $|A^{-1}|$ is equal to
 (a) $-\frac{1}{3}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{4}{3}$
71. For the system of equations
 $5x + 2y = 4; 7x + 3y = 5$
 the values of x and y are respectively
 (a) $x = 2, y = -3$ (b) $x = 2, y = 3$
 (c) $x = -2, y = -3$ (d) $x = -2, y = 3$
72. If the system of equations
 $4x - 3y = 3; 3x - 5y = 7$
 has unique solution, then
 (a) $x = \frac{6}{11}$ and $y = \frac{19}{11}$ (b) $x = -\frac{6}{11}, y = -\frac{19}{11}$
 (c) $x = -\frac{6}{11}$ and $y = \frac{19}{11}$ (d) $x =$ and $y = -\frac{9}{11}$

ANSWERS

1. (a) 2. (a) 3. (b) 4. (a) 5. (b) 6. (c) 7. (a) 8. (b) 9. (b) 10. (b) 11. (b) 12. (d) 13. (a)
 14. (a) 15. (a) 16. (a) 17. (a) 18. (b) 19. (b) 20. (a) 21. (a) 22. (c) 23. (b) 24. (b) 25. (d) 26.
 (d) 27. (b) 28. (b) 29. (c) 30. (a) 31. (d) 32. (a) 33. (a) 34. (b) 35. (a) 36. (a) 37. (a) 38. (a) 39.
 (b) 40. (a) 41. (a) 42. (c) 43. (a) 44. (d) 45. (c) 46. (a) 47. (c) 48. a) 49. (a) 50. (d) 51. (a) 52.
 (c) 53. (c) 54. (a) 55. (b) 56. (a) 57. (a) 58. (c) 59. (c) 60. (a) 61. (b) 62. (d) 63. (c) 64. (b) 65.
 (a) 66. (a) 67. (a) 68. (c) 69. (c) 70. (a) 32. (b) 71. (a) 72. (b)

(SHORT/LONG ANSWER QUESTIONS)

- If A is an $m \times n$ matrix and B is $n \times p$ matrix does AB exist? If yes, write its order.
- If $A = \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$. Write the orders of AB and BA.
- If $A = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$, write AB.
- If $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, write AA^T .
- Give an example of two non-zero 2×2 matrices A and B such that $AB = O$.

6. If $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$, find $A + A^T$.
7. If the matrix $A = \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$ is skew-symmetric, find the values of 'a' and 'b'.
8. Let A and B be matrices of orders 3×2 and 2×4 respectively. Write the order of matrix AB.
9. If $A = \begin{bmatrix} 3 & 5 \\ 7 & 9 \end{bmatrix}$ is written as $A = P + O$, where as $A = P + Q$, where P is symmetric and Q is skew-symmetric matrix, then write the matrix P.
10. If $\begin{bmatrix} 1 & 0 \\ y & 5 \end{bmatrix} + 2 \begin{bmatrix} x & 0 \\ 1 & -2 \end{bmatrix} = I$, where I is 2×2 unit matrix. Find x and y.
11. If $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, satisfies the matrix equation $A^2 = kA$, write the value of k.
12. If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ satisfies $A^4 = \lambda A$, then write the value of λ
13. If $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$, find A^2 .
14. If $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$, find A^3 .
15. If $A = \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$, find A^4 .
16. If $[x \ 2] \begin{bmatrix} 3 \\ 4 \end{bmatrix} = 2$, find x
17. If $A = [a_{ij}]$ is a 2×2 matrix such that $a_{ij} = i + 2j$, write A.
18. Write matrix A satisfying $A + \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ -3 & 8 \end{bmatrix}$.
19. If $A = [a_{ij}]$ is a square matrix such that $a_{ij} = i^2 - j^2$, then write whether A is symmetric or skew-symmetric.
20. For any square matrix write whether AA^T is symmetric or skew-symmetric.
21. If $A = [a_{ij}]$ is a skew-symmetric matrix, then write the value of $\sum_i a_{ii}$.
22. If $A = [a_{ij}]$ is a skew-symmetric matrix, then write the value of $\sum_i \sum_j a_{ij}$.
23. If A and B are symmetric matrices, then write the condition for which AB is also symmetric.
24. If B is a skew-symmetric matrix, write whether the matrix $AB A^T$ is symmetric or skew-symmetric.
25. If B is a symmetric matrix, write whether the matrix $AB A^T$ is symmetric or skew-symmetric.
26. If A is a skew-symmetric and $n \in \mathbb{N}$ such that $(A^n)^T = \lambda A^n$, write the value of λ .
27. If A is a symmetric matrix and $n \in \mathbb{N}$, write whether A^n is symmetric or skew-symmetric or neither of these two.
28. If A is a skew-symmetric matrix and n is an even natural number, write whether A^n is symmetric or skew-symmetric or neither of these two.
29. If A is a skew-symmetric matrix and n is an odd natural number, write whether A^n is symmetric or skew-symmetric or neither of the two.
30. If A and B are symmetric matrices of the same order, write whether $AB - BA$ is symmetric or skew-symmetric or neither of the two.
31. Write a square matrix which is both symmetric as well as skew-symmetric.
32. Find the values of x and y, if $2 \begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$.

33. If $\begin{bmatrix} x+3 & 4 \\ y-4 & x+y \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 3 & 9 \end{bmatrix}$, find x and y .
34. Find the value of x from the following: $\begin{bmatrix} 2x-y & 5 \\ 3 & y \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 3 & -2 \end{bmatrix}$.
35. Find the value of y , if $\begin{bmatrix} x-y & 2 \\ x & 5 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 3 & 5 \end{bmatrix}$.
36. Find the value of x , if $\begin{bmatrix} 3x+y & -y \\ 2y-x & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -5 & 3 \end{bmatrix}$.
37. If matrix $A = [1 \ 2 \ 3]$, write AA^T .
38. If $\begin{bmatrix} 2x+y & 3y \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 6 & 4 \end{bmatrix}$, then find x .
39. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, find $A + A^T$.
40. If $\begin{bmatrix} a+b & 2 \\ 5 & b \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 2 & 2 \end{bmatrix}$, then find a .
41. If A is a matrix of order 3×4 and B is a matrix of order 4×3 , find the order of the matrix of AB .
42. If $[2 \ 1 \ 3] \begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = A$, then write the order of matrix A .
43. If $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ k & 23 \end{bmatrix}$, then write the value of k .
44. If I is the identity matrix and A is a square matrix such that $A^2 = A$, then what is the value $(I + A)^2 - 3A$?
45. If $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ is written as $B + C$, where B is a symmetric matrix and C is a skew-symmetric matrix, then find B .
46. If A is 2×3 matrix and B is a matrix such that $A^T B$ and BA^T both are defined, then what is the order of B ?
47. What is the total number of 2×2 matrices with each entry 0 or 1?
48. If $\begin{bmatrix} x & x-y \\ 2x+y & 7 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 8 & 7 \end{bmatrix}$, then find the value of y .
49. If a matrix has 5 elements, write all possible orders it can have.
50. For a 2×2 matrix $A = [a_{ij}]$ whose elements are given by $a_{ij} = \frac{i}{j}$, write the value of a_{12} .
51. If $x \begin{bmatrix} 2 \\ 3 \end{bmatrix} + y \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$, find the value of x .
52. If $\begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} = A + \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$, then find matrix A .
53. If $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$, find the value of b .
54. For what value of x , is the matrix $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$ a skew-symmetric matrix?
55. If matrix $A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$ and $A^2 = pA$, then write the value of p .
56. If A is a square matrix such that $A^2 = A$, then write the value of $7A - (I + A)^3$, where I is the identity matrix.
57. If $2 \begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$, find $x - y$.
58. If $[x \ 1] \begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = 0$, find x .
59. If $\begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix} = \begin{bmatrix} 2a+2 & b+2 \\ 8 & a-8b \end{bmatrix}$, write the value of $a - 2b$.

60. Write a 2×2 matrix which is both symmetric and skew-symmetric.
61. If $\begin{bmatrix} xy & 4 \\ z+6 & x+y \end{bmatrix} = \begin{bmatrix} 8 & w \\ 0 & 6 \end{bmatrix}$, write the value of $(x + y + z)$.
62. Construct a 2×2 matrix $A = [a_{ij}]$ whose elements a_{ij} are given by $a_{ij} = \begin{cases} \frac{|-3i+j|}{2}, & \text{if } i \neq j \\ (i + j)^2, & \text{if } i = j \end{cases}$
63. If $\begin{bmatrix} x+y \\ x-y \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, then write the value of (x, y) .
64. Matrix $A = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$ is given to be symmetric, find the values of a and b .
65. Write the number of all possible matrices of order 2×2 with each entry 1, 2 or 3.
66. If A is a singular matrix, then write the value of $|A|$.
67. For What value of x , the matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ is singular?
68. Write the value of the determinant $\begin{vmatrix} 2 & 3 & 4 \\ 2x & 3x & 4x \\ 5 & 6 & 6 \end{vmatrix}$.
69. State whether the matrix $\begin{bmatrix} 2 & 3 \\ 6 & 4 \end{bmatrix}$ is singular or nonsingular.
70. If $A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$, find $|AB|$.
71. If $A = [a_{ij}]$ is a 3×3 diagonal matrix such that $a_{11} = 1$, $a_{22} = 2$ and $a_{33} = 3$, then find $|A|$.
72. If $A = [a_{ij}]$ is a 3×3 scalar matrix such that $a_{11} = 2$, then write the value of $|A|$.
73. If I_3 denotes identity matrix of order 3×3 , write the value of its determinant.
74. If the matrix $\begin{bmatrix} 5x & 2 \\ -10 & 1 \end{bmatrix}$ is singular, find the value of x .
75. Write the cofactor of a_{12} in the matrix $\begin{bmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{bmatrix}$.
76. If $\begin{vmatrix} 2x+5 & 3 \\ 5x+2 & 9 \end{vmatrix} = 0$, find x .
77. Find the value of x from the following: $\begin{vmatrix} x & 4 \\ 2 & 2x \end{vmatrix} = 0$
78. Write the value of the determinant $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$
79. What is the value of the determinant $\begin{vmatrix} 0 & 2 & 0 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{vmatrix}$?
80. For what value of x is the matrix $\begin{bmatrix} 6-x & 4 \\ 3-x & 1 \end{bmatrix}$ singular?
81. If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ are two square matrices, find AB and hence solve the system of linear equations:
 $x - y = 3, 2x + 3y + 4z = 17, y + 2z = 7$
82. If $A = \begin{bmatrix} 2 & 3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1} and hence solve the system of linear equations:
 $2x - 3y + 5z = 11, 3x + 2y - 4z = -5, x + y - 2z = -3$

83. (i) Given $A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & 4 \\ 2 & -1 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$, find BA and use this to solve the system of equations $y + 2z = 7$, $x - y = 3$, $2x + 3y + 4z = 17$

(ii) If $A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 2 \\ -3 & 1 & -1 \end{bmatrix}$, find A^{-1} and hence solve the system of equations $2x + y - 3z = 13$, $3x + 2y + z = 4$, $x + 2y - z = 8$.

(iii) Use the product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 1 & 1 & -2 \end{bmatrix}$ to solve the system of equations $x + 3z = -9$, $-x + 2y - 2z = 4$, $2x - 3y + 4z = -3$.

84. The sum of three numbers is 2. If twice the second number is added to the sum of first and third, the sum is 1. By adding second and third number to five times the first number, we get 6. Find the three numbers by using matrices.

85. An amount of Rs. 10,000 is put into three investments at the rate of 10, 12 and 15% per annum. The combined income is Rs. 1310 and the combined income of first and second investment is Rs. 190 short of the income from the third. Find the investment in each using matrix method.

86. A company produces three products every day. Their production on a certain day is 45 tons. It is found that the production of third product exceeds the production of first product by 8 tons while the total production of first and third product is twice the production of second product. Determine the production level of each product using matrix method.

87. The prices of three commodities P, Q and R are Rs. x , y and 2 per unit respectively. A purchases 4 units of R and sells 3 units of P and 5 units of Q. B purchases 3 units of Q and sells 2 units of P and 1 unit of R. C purchases 1 unit of P and sells 4 units of Q and 6 units of R. In the process A, B and C earn Rs. 6000, Rs. 5000 and Rs. 13000 respectively. If selling the units is positive earning and buying the units is negative earnings, find the price per unit of three commodities by using matrix method.

88. Two factories decided to award their employees for three values of (a) adaptable to new techniques, (b) careful and alert in difficult situations and (c) keeping calm in tense situations, at the rate of Rs. x , Rs. y and Rs. z per person respectively. The first factory decided to honour respectively 2, 4 and 3 employees with a total prize money of Rs. 29000. The second factory decided to honour respectively 5, 2 and 3 employees with the prize money of Rs. 30500. If the three prizes per person together cost Rs. 9500, then

(i) represent the above situation by a matrix equation and form linear equations using matrix multiplication. 3

(ii) Solve these equations using matrices.

89. A total amount of Rs. 7000 is deposited in three different saving bank accounts with annual interest rates 5%, 8% and $8\frac{1}{2}\%$ respectively. The total annual interest from these three accounts is Rs. 550. Equal amounts have been deposited in the 5% and 8% savings accounts. Find the amount deposited in each of the three accounts, with the help of matrices.

90. A shopkeeper has 3 varieties of pens 'A', 'B' and 'C'. Meenu purchased 1 pen of each variety for a total of Rs. 21. Jeenu purchased 4 pens of 'A' variety, 3 pens of 'B' variety and 2 pens of 'C' variety for Rs. 60. While Shikha purchased 6 pens of 'A' variety, 2 pens of 'B' variety and 3 pens of 'C' variety for Rs. 70. Using matrix method find the cost of each pen.

91. Puransh has invested a part of his investment in 10% bond A and a part in 15% bond B. His interest income during first year is Rs. 4000. If he invests 20% more in 10% bond A and 10% more in 15% bond B his income during second year increases by Rs. 500. Find his initial investments and new investments in bonds A and B using matrix method.

92. To control a crop disease it is necessary to use 8 units of chemical A, 14 units of chemical B and 13 units of chemical C. One barrel of spray P contains one unit of A, 2 units of B and 3 units of C. One barrel of spray Q contains 2 units of A, 3 units of B and 2 units of C. One barrel of spray R contains 1 unit of A, 2 units of B and 2 units of C. Using matrix method, find how many barrels of each spray be used to just meet the requirement?

93. A firm produces two products P_1 and P_2 passing through two machines M_1 and M_2 before completion. M_1 can produce either 10 units of P_1 or 15 units of P_2 per hour. M_2 can produce 15 units of either product per hour. Find daily production of P_1 and P_2 if time available is 12 hours on machine M_1 and 10 hours on M_2 per day using matrix inversion method.

94. Tire equilibrium condition for three related markets is given by

$$11p_1 - p_2 - p_3 = 31$$

$$-p_1 + 6p_2 - 2p_3 = 26$$

$$-p_1 - 2p_2 + 7p_3 = 24$$

Using matrix inversion method, find the equilibrium prices of each market.

95 Express the following matrices as the sum of symmetric and skew - symmetric matrices:

$$\begin{bmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{bmatrix}$$

ANSWERS

1. Yes, $m \times p$ 2. 2×2 and 3×3 3. $\begin{bmatrix} -7 \\ 2 \end{bmatrix}$ 4. $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$

5. $A = \begin{bmatrix} 2 & 0 \\ 3 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ 2 & -1 \end{bmatrix}$ 6. $\begin{bmatrix} 4 & 8 \\ 8 & 14 \end{bmatrix}$ 7. $a = -2$, $b = 3$

8. 3×4 9. $\begin{bmatrix} 3 & 6 \\ 6 & 9 \end{bmatrix}$ 10. $x = 0$, $y = -2$ 11. 2

12. 8 13. $-A$ or, I_3 14. A 15. $\begin{bmatrix} 81 & 0 \\ 0 & 81 \end{bmatrix}$

16. -2 17. $\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$ 18. $\begin{bmatrix} 1 & -9 \\ -2 & 4 \end{bmatrix}$ 19. skew-symmetric

20. symmetric 21. 0 22. 0 23. $AB = BA$

24. skew-symmetric 25. Symmetric 26. $(-1)^n$ 27. symmetric

28. symmetric 29. skew-symmetric 30. skew-symmetric 31. null matrix

32. $x = 3$, $y = 3$ 33. $x = 2$, $y = 7$ 34. $x = 2$ 35. $y = 1$

36. $x = 1$ 37. 14 38. $x = 3$, $y = 0$ 39. $\begin{bmatrix} 2 & 5 \\ 5 & 8 \end{bmatrix}$

40. 4 41. 3×3 42. 1×1 43. 17

44. I 45. $\begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}$ 46. 2×3 47. 16

48. 2 49. 1×5 , 5×1 50. $1/2$ 51. $x = 3$

52. $\begin{bmatrix} 8 & -3 & 5 \\ -2 & -3 & -6 \end{bmatrix}$ 53. $b = 2$ 54. $x = 2$ 55. 4 56. $-I$

57. $x = 2$, $y = -8$ 58. $x = 2$ 59. 0 60. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ 61. 0

62. $\begin{bmatrix} 4 & 1/2 \\ 5/2 & 16 \end{bmatrix}$ 63. $(-1, 1)$ 64. $a = -\frac{2}{3}$, $b = \frac{3}{2}$ 65. $3^4 = 81$

66. 0 67. 3 68. 0 69. Non-singular 70. 0 71. 6 72. 8 73. 1

74. -4 75. 46 76. -13 77. ± 2 78. 0 79. 8 80. 2

81. $x = 2, y = -1, z = 4$ 82. $x = 1, y = 2, z = 3$
 83 (i) $x = 2, y = -1, z = 4$ (ii) $x = 1, y = 2, z = -3$ (iii) $x = 36, y = 5, z = -15$
 84. 1, -1, 2 85. Rs. 2000, Rs. 3000, Rs. 5000
 86. 11, 15, 19 87. $x = 3000, y = 1000, z = 2000$
 88. $x = 2500, y = 3000, z = 4000$ 89. Rs. 1125, Rs. 1125, Rs. 4750
 90. Variety A : Rs. 5, Variety B: Rs. 8, Variety C : Rs. 8
 91. Initial investments: Rs. 10,000 in A; Rs. 20,000 in B
 New investments; Rs. 12,000 in A; Rs. 22,000 in B
 92. P : 1 Barrel; Q : 2 Barrels; R : 3 Barrels 93. P_1 : 60 units; P_2 : 90 units
 94. $p_1 = 4, p_2 = 7, p_3 = 6$

(CASE STUDY/SOURCE BASED QUESTIONS)

96 **Read the following text carefully and answer the questions that follow:**

A trust invested some money in two type of bonds. The first bond pays 10% interest and second bond pays 12% interest. The trust received ₹ 2800 as interest. However, if trust had interchanged money in bonds, they would have got ₹ 100 less as interest.

Let the amount invested in first type and second type of bonds be ₹ x and ₹ y respectively.

- Write the equations in terms of x and y . (1)
- If $A = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$, then find A^3 . (1)
- Find the amount invested by trust in first and second bonds. (2)

OR

if A is the matrix formed by coefficients of x and y of the given situation, then find the value of $|\text{adj } A|$. (2)

97 **Read the following text carefully and answer the questions that follow:**

To promote the making of toilets for women, an organisation tried to generate awareness through

- house calls
- emails and
- announcements.



The cost for each mode per attempt is given below:

- ₹ 50
- ₹ 20
- ₹ 40

The number of attempts made in the villages X, Y and Z are given below:

	(i)	(ii)	(iii)
X	400	300	100
Y	300	250	75
Z	500	400	150

Also, the chance of making of toilets corresponding to one attempt of given modes is

- 2%
- 4%
- 20%

- What is the cost incurred by the organisation on village X? (1)

2. What is cost incurred by the organisation on village Y? (1)
3. What is the cost incurred by the organisation on village Z? (2)

OR

What are the total number of toilets that can be expected after the promotion in village X?
(2)

98 **Read the following text carefully and answer the questions that follow:**

Raja purchases 3 pens, 2 pencils and 1 mathematics instrument box and pays ₹ 41 to the shopkeeper. His friends, Daya and Anil purchases 2 pens, 1 pencil, 2 instrument boxes and 2 pens, 2 pencils and 2 mathematical instrument boxes respectively. Daya and Anil pays ₹ 29 and ₹ 44 respectively. Based on the above information answer the following:

1. What will be the cost of one pen? (1)
2. What will be the cost of one pen and one pencil? (1)
3. What will be the cost of one pen and one mathematical instrument box? (2)

OR

What will be the cost of one pencil and one mathematical instrument box? (2)

99 **Read the following text carefully and answer the questions that follow:**

Three friends Ravi, Raju and Rohit were doing buying and selling of stationery items in a market. The price of per dozen of Pen, Notebook and toys are Rupees x , y and z respectively. Ravi purchases 4 dozens of notebooks and sells 2 dozens of pens and 5 dozens of toys. Raju purchases 2 dozens of toy and sells 3 dozens of pens and 1 dozen of notebooks. Rohit purchases one dozen of pens and sells 3 dozens of notebooks and one dozen of toys. In the process, Ravi, Raju and Rohit earn ₹ 1500, ₹ 100 and ₹ 400 respectively.



1. What is the price of one dozen of pens? (1)
2. What is the total price of one dozen of pen and one dozen of notebook? (1)
3. What is the sale amount of Ravi? (2)

OR

How much is the total amount of purchase made by all three friends? (2)

100 **Read the following text carefully and answer the questions that follow:**

Two schools Oxford and Navdeep want to award their selected students on the values of sincerity, truthfulness and helpfulness. Oxford wants to award ₹ x each, ₹ y each and ₹ z each for the three respective values to 3, 2 and 1 students respectively with a total award money of 1600. Navdeep wants to spend 2300 to award its 4, 1 and 3 students on the respective values (by giving the same amount to the three values as before). The total amount of the award for one prize on each is ₹ 900.



1. What is the value of $x + y + z$? (1)

2. What is the value of $4x + y + 3z$? (1)
3. What is the value of y ? (2)

OR

What is the value of $2x + 3y$? (2)

101 Read the following text carefully and answer the questions that follow:

Two schools P and Q decided to award their selected students for the values of discipline and honesty in the form of prizes at the rate of ₹ x and ₹ y respectively. School P decided to award respectively 3, 2 students a total prize money of ₹ 2300 and school Q decided to award respectively 5, 3 students a total prize money of ₹ 3700.

1. Write the matrix equation representing the above situation. (1)
2. Find the value of the determinant of coefficients of x and y . (1)
3. Find the values of x and y respectively (use Cramer's rule). (2)

OR

Find the inverse of matrix A. (2)

102 Read the following text carefully and answer the questions that follow:

On his birthday, Ishan decided to donate some money to children of an orphanage home. If there were 8 children less, every one would have got ₹ 10 more. However, if there were 16 children more, every one would have got ₹ 10 less.

Let the number of children be x and amount distributed by Ishan to each child be ₹ y .

1. Write matrix equation represent the information given above. (1)
2. If A is the coefficient matrix of above situation, then what is $(\text{adj } A)$? (1)
3. Find the number of children and amount donated by Ishan. (2)

OR

If the coefficient matrix A satisfy the following matrix equation $A^2 - kA + 20I = 0$, then find k . (2)

103 Read the following text carefully and answer the questions that follow:

Two farmers Ramakishan and Gurucharan Singh cultivate only three varieties of rice namely Basmati, Permal and Naura. The sale (in rupees) of these varieties of rice by both the farmers in the month of September and October are given by the following matrices A and B.



September sales (in Rupees)

$$A = \begin{bmatrix} 10,000 & 20,000 & 30,000 \\ 50,000 & 30,000 & 10,000 \end{bmatrix} \begin{matrix} \text{Ramakishan} \\ \text{Gurucharan} \end{matrix}$$

October sales (in Rupees)

$$B = \begin{bmatrix} 5,000 & 10,000 & 6,000 \\ 20,000 & 10,000 & 10,000 \end{bmatrix} \begin{matrix} \text{Ramakishan} \\ \text{Gurucharan} \end{matrix}$$

Based on the information given above, answer the following questions:

1. The total sales in September and October for each farmer in each variety can be represented as _____. (1)
2. What is the value of A_{23} ? (1)
3. If Ramkishan receives 2% profit on gross sales, compute his profit for each variety sold in October. (2)

104 Read the following text carefully and answer the questions that follow:

The management committee of a residential colony decided to award some of its members (say x) for honesty, some (say y) for helping others and some others (say z) for supervising the workers to keep the colony neat and clean. The sum of all the awardees is 12. Three times the sum of awardees for cooperation and supervision added to two times the number of awardees for honesty is 33. The sum of the number of awardees for honesty and supervision is twice the number of awardees for helping.



1. What is the value of $x + y + z$? (1)
2. What is the value of $x - 2y$? (1)
3. What is the value of z ? (2)

OR

What is the value of $x + 2y$? (2)

ANSWERS

96. (i) $5x + 6y = 140000$, $6x + 5y = 135000$ (ii) $\begin{bmatrix} 125 & 0 \\ 0 & 125 \end{bmatrix}$ (iii) $x = 10000$, $y = 15000$ or -11
- ii. 97. (i) 30000 (ii) 23000 (iii) 39000 or 40 98. (i) 2 (ii) 17 (iii) 7 or 20 99. (i) 100 (ii) 300 (iii) 1200 or 1500 100. (i) 900 (ii) 2300 (iii) 300 or 1300 101. (i) $\begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2300 \\ 3700 \end{bmatrix}$ (ii) -1 (iii) $x = 500$ $y = 400$ or $\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}$ 102. (i) $5x - 4y = 80$, $5x + 8y = 80$ (ii) $\begin{bmatrix} 8 & 4 \\ 5 & 5 \end{bmatrix}$ (iii) 960 or $k = 13$ 103. i. A + B (ii) 10000 (iii) ₹100, ₹ 200 and ₹120 **OR** ₹1000, ₹600, ₹200 104. i. 12 (ii)-z (iii) 5 **OR** 11

DIFFERENTIATION & ITS APPLICATION

1. If $xy = 2$, then $\frac{dy}{dx}$ is equal to
 (a) $\frac{y}{x}$ (b) $-\frac{x}{y}$ (c) $\frac{x}{y}$ (d) $-\frac{y}{x}$
2. For the curve $\sqrt{x} + \sqrt{y} = 1$, $\frac{dy}{dx}$ at $(\frac{1}{4}, \frac{1}{4})$ is
 (a) -2 (b) 0 (c) -1 (d) 2
3. If $x^2 + xy + y^2 = 1$, then $\frac{dy}{dx}$ is equal to
 (a) $\frac{-(2x+y)}{x+2y}$ (b) $\frac{(2x+y)}{x+2y}$ (c) $\frac{-(x+2y)}{(2x+y)}$ (d) $\frac{x+y}{2x-y}$
4. If $x^3 + 3xy = y$, then $\frac{dy}{dx}$ is equal to
 (a) $\frac{3(x^2+y)}{1-3x}$ (b) $\frac{3(x^2+y^2)}{1+3x}$ (c) $\frac{x^2+y^2}{1-3x}$ (d) $\frac{x^2+y^2}{1+3x}$
5. If $(x^2 + y^2) = xy$, then $\frac{dy}{dx}$ is equal to
 (a) $\frac{y-2x}{2y-x}$ (b) $\frac{y-2x}{y-x}$ (c) $\frac{y+2x}{2y-x}$ (d) $\frac{y+x}{x-y}$

6. If $x^3 + 8xy + y^3 = 64$, then $\frac{dy}{dx}$ is equal to
 (a) $\frac{-(3x^2+8y)}{8x+3y^3}$ (b) $\frac{3x^2+8y}{8x+3y^3}$ (c) $\frac{3x+8y^2}{8x^2+3y}$ (d) None of these
7. If $x^2 + y^2 = 4xy$, then $\frac{dy}{dx}$ is equal to
 (a) $\frac{2y-x}{y-2x}$ (b) $\frac{2y+x}{y+2x}$ (c) $\frac{2y-x}{y+2x}$ (d) $\frac{y-x}{y+2x}$
8. If $xy + xy^2 = x$, then $\frac{dy}{dx}$ at $x = 1$ and $y = -1$ is equal to
 (a) 1 (b) -1 (c) 0 (d) 5
9. If $x = at^2$ and $y = 2at$, then $\frac{dy}{dx}$ is equal to
 (a) t (b) $\frac{1}{t}$ (c) $\frac{-1}{t^2}$ (d) None of these
10. If $x = 8t$ and $y = \frac{8}{t}$, then $\frac{dy}{dx}$ is equal to
 (a) $\frac{-1}{t^2}$ (b) $\frac{1}{t^2}$ (c) $\frac{1}{t}$ (d) $\frac{-1}{t}$
11. If $x = 2at^2$ and $y = at^4$, then $\frac{dy}{dx}$ is equal to
 (a) t (b) t^2 (c) t^3 (d) t^4
12. If $x = \log t^2$ and $y = e^{2t}$, then $\frac{dy}{dx}$ is equal to
 (a) te^t (b) te^{2t} (c) $-te^t$ (d) $-te^{-t}$
13. If $x = t^3 + t^2$ and $y = t^2 + t$, then $\frac{dy}{dx}$ at $t = 1$ is equal to
 (a) $\frac{3}{5}$ (b) $-\frac{3}{5}$ (c) $\frac{2}{5}$ (d) $-\frac{2}{5}$
14. If $x = t^2$ and $y = t^3$, then $\frac{d^2y}{dx^2}$ is equal to
 (a) $\frac{3}{2}$ (b) $\frac{3t}{4}$ (c) $\frac{3t}{2}$ (d) $\frac{3}{4}$
15. If $x = t^5$ and $y = t^6$, then $\frac{dy}{dx}$ at $t = \frac{5}{6}$ is equal to
 (a) 4 (b) 6 (c) 1 (d) -1
16. If $x = 2t^2$ and $y = 8t^3$, then $\frac{dy}{dx}$ at $t = \frac{1}{6}$ is equal to
 (a) 1 (b) 0 (c) 8 (d) -1
17. If $x = 40t$, $y = 20t^2$, then $\frac{dy}{dx}$ is equal to
 (a) $-t$ (b) t (c) $2t$ (d) $-2t$
18. If $x = t^2 + 1$ and $y = t + 5$, then $\frac{dy}{dx}$ at $t = \frac{1}{2}$ is equal to
 (a) -1 (b) 0 (c) 2 (d) 1
19. If $x = \log(t^2 + t + 1)$ and $y = 2\log t$, then $\frac{dy}{dx}$ at $t = -1$ is equal to
 (a) 1 (b) 2 (c) 3 (d) 4
20. If $x = e^t$ and $y = te^t$, then $\frac{dy}{dx}$ at $t = -1$ is equal to

- (a) 1 (b) 0 (c) -1 (d) 2
21. If $x = e^t$ and $y = 8 + e^{2t}$, then $\frac{dy}{dx}$ at $t = 1$ is equal to
 (a) $2e$ (b) e (c) e^{-1} (d) $\frac{e}{2}$
22. If $y = e^{ax}$, then $\frac{d^2y}{dx^2}$ is equal to
 (a) a^2e^{ax} (b) e^{ax} (c) ae^{ax} (d) a^2e^x
23. If $y = Ae^{5x} + Be^{-5x}$, then $\frac{d^2y}{dx^2}$ is equal to
 (a) $25y$ (b) $5y$ (c) $-25y$ (d) $15y$
24. If $y = \frac{\ln x}{x}$, then $\frac{d^2y}{dx^2}$ is equal to
 (a) $\frac{2 \ln x + 3}{x^3}$ (b) $\frac{2 \ln x - 3}{x^3}$
 (c) $\frac{\ln x + 3}{x^3}$ (d) None of the above
25. If $y = (x^2 + x + 3)^2$, then $\frac{d^2y}{dx^2}$ at $x = -1$ is equal to
 (a) 12 (b) 26 (c) 14 (d) -14
26. If $y = (x + 1)^3 + (x + 2)^2$, then $\frac{d^3y}{dx^3}$ is equal to
 (a) 1 (b) 2 (c) 6 (d) 5
27. If $y = e^x + e^{-x}$ and $\frac{dy}{dx} = k \frac{d^3y}{dx^3}$, then k is equal to
 (a) 1 (b) 3 (c) 4 (d) 2
28. If $y = \log x^2$, then $\frac{d^2y}{dx^2}$ is equal to
 (a) $\frac{4}{x^3}$ (b) $\frac{-4}{x^3}$ (c) $\frac{2}{x^3}$ (d) $\frac{-2}{x^3}$
29. If $y = 5x^4 + 3x^3 + 2x^2 + x + 8$, then $\frac{d^4y}{dx^4}$ is equal to
 (a) -120 (b) 120 (c) 110 (d) 60

Assertion-Reasoning MCQs

Directions (Q. Nos. 30-34) Each of these questions contains two statements Assertion (A) and Reason (R). Each of the questions has four alternative choices, any one of the which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

- (a) A is true, R is true; R is a correct explanation of A.
 (b) A is true, R is true; R is not a correct explanation of A.
 (c) A is true; R is false
 (d) A is false; R is true.

30. **Assertion (A)** If $xy = e^{x-y}$, then $\frac{dy}{dx} = \frac{y(x-1)}{x(1+y)}$.

Reason (R) $\frac{d}{dx}(u \cdot v) = u \frac{d}{dx}v + v \frac{d}{dx}u$

31. **Assertion (A)** If $x = \frac{2at}{1+t^3}$ and $y = \frac{2at^2}{(1+t^3)^2}$, then $\frac{dy}{dx} = \frac{x}{a}$.

- Reason (R)** If $x = f(t)$ and $y = g(t)$, then $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$.
32. **Assertion (A)** If $f(x) = \log x$, then $f''(x) = -\frac{1}{x^2}$.
- Reason (R)** If $y = x^3 \log x$, then $\frac{d^2y}{dx^2} = x(5 + 6 \log x)$.
33. **Assertion (A)** If $y = 500e^{7x} + 600e^{-7x}$, then $\frac{d^2y}{dx^2} = 49y$.
- Reason (R)** If $e^y(x+1) = 1$, then $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$.
34. **Assertion (A)** If $x = t$ and $y = t^2$, then $\frac{d^2y}{dx^2}$ is 2.
- Reason (R)** Let $f(x) = x^{2n}$, $n \in \mathbb{N}$, Then, $f^n(x)$, i.e. n th derivative of $f(x)$ w.r.t. x is $\frac{2n!}{n!} x^n$.
35. The function $f(x) = 9x + 8$, $x \in \mathbb{R}$ is an increasing function.
 (a) True (b) False
 (c) Can't say (d) Partially True/False
36. The function $f(x) = x^2$ is strictly decreasing in the interval
 (a) $(-\infty, 0)$ (b) $(0, \infty)$
 (c) $(1, 4)$ (d) None of these
37. If $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = 2x + \cos x$, then f
 (a) both increasing and decreasing
 (b) neither increasing nor decreasing
 (c) is a decreasing function
 (d) is an increasing function
38. $f(x) = 2x^3 - 9x^2 + 12x - 3$ is increasing
 (a) inside the interval $(1, 2)$ (b) inside the interval $(2, 3)$
 (c) outside the interval $(1, 2)$ (d) outside the interval $(2, 3)$
39. Which of the following is true for the function $f(x) = 9x - 5$?
 (a) $f(x)$ is strictly increasing on \mathbb{R} (b) $f(x)$ is strictly decreasing on \mathbb{R}
 (c) Both (a) and (b) are false (d) $f(x)$ is decreasing on \mathbb{R}
40. The function given $f(x) = e^{2x}$ is ...A... on \mathbb{R} . Here, A refers to
 (a) strictly increasing (b) strictly decreasing
 (c) neither increasing nor decreasing (d) decreasing
41. The function given by $f(x) = 3 - 7x$ is strictly decreasing on \mathbb{R} .
 (a) True (b) False
 (c) Can't say (d) Partially true
42. The values of x for which $y = [x(x-2)]^2$ is an increasing function, are
 (a) $0 < x < 1$ and $x > 2$ (b) $0 < x < 2$ and $x > 3$

- (c) $0 < x < \frac{1}{2}$ and $x > \frac{3}{2}$ (d) $0 < x < \frac{3}{2}$ and $x > 4$
43. If $y = x(x - 3)^2$ decreases for the values of x given by
 (a) $1 < x < 3$ (b) $x < 0$
 (c) $x > 0$ (d) $0 < x < \frac{3}{2}$
44. The function f given by $f(x) = 2x^2 - 3x$, is
 (a) strictly increasing in the interval $(\frac{3}{4}, \infty)$
 (b) strictly decreasing in the interval $(-\infty, \frac{3}{4})$
 (c) Both (a) and (b) are true
 (d) Both (a) and (b) are false
45. The function $f(x) = 4x^3 - 18x^2 + 27x - 7$ is always on \mathbb{R}
 (a) increasing (b) decreasing
 (c) strictly decreasing (d) None of these
46. For $x \in \mathbb{R}$, the function $f(x) = x^3 - 6x^2 + 12x - 18$ is
 (a) an increasing function (b) decreasing function
 (c) both (a) and (b) are true (d) None of the above
47. If the function f be given by $f(x) = xe^{1-x}$, then choose the correct statement from the following.
 (a) f is strictly increasing in $(\frac{1}{2}, 2)$
 (b) f is decreasing in $(0, 2)$
 (c) f is increasing in $(0, \infty)$
 (d) f is strictly decreasing in $(1, \infty)$
48. The function $f(x) = \frac{5}{x} - 9$ is of which nature for $x \in \mathbb{R}$, ($x \neq 0$).
 (a) increasing (b) decreasing
 (c) strictly increasing (d) strictly decreasing
49. The critical points for the function $f(x) = x^3 - 2x^2 + x + 1$ are
 (a) $1, \frac{1}{3}$ (b) $1, 0$ (c) $2, 1$ (d) $4, 3$
50. The function $f(x) = 2x^3 - 3x^2 - 12x + 4$, has
 (a) two points of local maximum (b) two points of local minimum
 (c) one maxima and one minima (d) neither maxima nor minima
51. The maximum slope of curve
 $y = -x^3 + 3x^2 + 9x - 27$ is
 (a) 0 (b) 12 (c) 16 (d) 32
52. If the function $f(x) = x^3 - 24x^2 + kx - 8$ is maximum at $x = 2$, then the value of k is
 (a) 80 (b) 86 (c) 84 (d) None of these

53. The minimum value of the function $f(x) = 2x^3 - 3x^2$ exists at x equals to
 (a) 0 (b) 2 (c) -1 (d) 1
54. The maximum value of $\left(\frac{1}{x}\right)^x$ is
 (a) e (b) e^e (c) $e^{1/e}$ (d) $\left(\frac{1}{e}\right)^{1/e}$
55. The absolute maximum values of a function f given by
 $f(x) = 12x^{4/3} - 6x^{1/3}$, $x \in [-1, 1]$.
 (a) 18 (b) 16 (c) 14 (d) 12
56. The total cost $C(x)$ of producing x items in a firm is given by
 $C(x) = 0.005x^3 - 0.02x^2 + 30x + 6000$.
 The marginal cost when 4 units are produced is
 (a) ₹ 29.08 (b) ₹ 28.08 (c) ₹ 31.08 (d) ₹ 30.08
57. The total revenue (in ₹) received from the sale of x units of a product is given by $R(x) = 3x^2 + 6x + 5$. The marginal revenue, when $x = 5$ is
 (a) ₹ 36 (b) ₹ 35 (c) ₹ 34 (d) ₹ 33

Assertion-Reasoning MCQs

Directions (Q. Nos. 36-44) Each of these questions contains two statements :

Assertion (A) and Reason (R). Each of these questions has four alternative choices in which any one of them is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

- (a) A is true, R is true; R is a correct explanation for A.
 (b) A is true, R is true; R is not a correct explanation for A.
 (c) A is true; R is false.
 (d) A is false; R is true.
58. **Assertion (A)** The function $f(x) = x^2 - 4x + 6$ is strictly increasing in the interval $(2, \infty)$.
Reason (R) The function $f(x) = x^2 - 4x + 6$ is strictly decreasing in the interval $(-\infty, 2)$.
59. **Assertion (A)** $y = \frac{e^x + e^{-x}}{2}$ is an increasing function on $[0, \infty)$.
Reason (R) $y = \frac{e^x - e^{-x}}{2}$ is an increasing function on $(-\infty, \infty)$
60. **Assertion (A)** If two positive numbers are such that sum is 16 and sum of their cubes is minimum, then numbers are 8, 8.
Reason (R) If f be a function defined on an interval I and $c \in I$ and let f be twice differentiable at c , then $x = c$ is a point of local minima, if $f'(c) = 0$ and $f''(c) > 0$ and $f(c)$ is local minimum value of f .
61. The function f be given by $f(x) = 2x^3 - 6x^2 + 6x + 5$.
Assertion (A) $x = 1$ is not a point of local maxima.

Reason (R) $x = 1$ is not a point of local minima.

62. Assertion (A) If x is real, then the minimum value of $x^2 - 8x + 17$ is 1.

Reason (R) If $f''(x) > 0$ at critical point, then the value of the function at critical point will be the minimum value of the function.

ANSWERS

Multiple Choice Questions

1. (d) 2. (c) 3. (a) 4. (a) 5. (a) 6. (a) 7. (a) 8. (b) 9. (b) 10. (a) 11. (b) 12. (b)
 13. (a) 14. (b) 15. (c) 16. (a) 17. (b) 18. (d) 19. (b) 20. (b) 21. (a) 22. (a) 23. (a) 24. (b) 25. (c)
 26. (c) 27. (a) 28. (a) 29. (b) 30. (b) 31. (a) 32. (b) 33. (b) 34. (b)
 35. (a) 36. (a) 37. (d) 38. (c) 39. (a) 40. (a) 41. (a) 42. (a) 43. (a) 44. (c) 45. (a) 46. (a) 47. (d)
 48. (b) 49. (a) 50. (c) 51. (b) 52. (c) 53. (d) 54. (c) 55. (a) 56. (d) 57. (a)
 58. (b) 59. (b) 60. (a) 61. (b) 62. (a)

(SHORT/LONG ANSWER QUESTIONS)

- Show that $f(x) = e^{2x}$ is increasing on \mathbb{R} .
- Show that $f(x) = e^{1/x}$, $x \neq 0$ is a decreasing function for all $x \neq 0$.
- Show that $f(x) = \log_a x$, $0 < a < 1$ is a decreasing function for all $x > 0$.
- Show that $f(x) = x^3 - 15x + 75x - 50$ is an increasing function for all $x \in \mathbb{R}$.
- Show that $f(x) = (x - 1)e^x + 1$ is an increasing function for all $x > 0$.
- Show that the function $x^2 - x + 1$ is neither increasing nor decreasing on $(0, 1)$.
- Show that $f(x) = x^9 + 4x^7 + 11$ is an increasing function for all $x \in \mathbb{R}$.
- Prove that the function $f(x) = x^3 - 6x^2 + 12x - 18$ is increasing on \mathbb{R} .
- State when a function $f(x)$ is said to be increasing on an interval $[a, b]$. Test whether the function $f(x) = x^2 - 6x + 3$ is increasing on the interval $[4, 6]$.
- Find the intervals in which $f(x) = \log_e(1 + x) - \frac{x}{1 + x}$ is increasing or decreasing.
- Find the intervals in which $f(x) = (x + 2)e^{-x}$ is increasing or decreasing.
- Show that the function f given by $f(x) = 10^x$ is increasing for all x .
- Prove that the function f given by $f(x) = x - [x]$ is increasing in $(0, 1)$.
- Prove that the following functions are increasing on \mathbb{R} :
 (i) $f(x) = 3x^5 + 40x^3 + 240x$ (ii) $f(x) = 4x^3 - 18x^2 + 27x - 27$
- Prove that the function f given by $f(x) = x^3 - 3x^2 + 4x$ is strictly increasing on \mathbb{R} .
- Find the value(s) of a for which $f(x) = x^3 - ax$ is an increasing function on \mathbb{R} .
- The cost and demand functions of a commodity are given by $C(x) = 75 + 2x$ and $p(x) = 85 - x$. Find (i) the revenue function (ii) the profit function.
- The demand function of a certain commodity is given by $p = 1000 - 25x + x^2$, where $0 \leq x \leq 20$. Find the price per unit and total revenue from the sale of (i) 2 units (ii) 5 units.
- A demand function for a commodity is known to be a linear function. The price for 5 units is Rs. 9 a unit and the price for 10 units is Rs. 12 per unit. Find the demand and revenue functions and determine the total revenue when 50 units are sold.
- A company sells x tins of talcum powder each day at Rs. 10 per tin. The cost of manufacturing is Rs. 6 per tin and the distributor charges Rs. 1 per tin. Besides these the daily overhead cost comes to Rs. 600. Determine the profit function. What is the profit if 500 tins are manufactured and sold in a day? How do you interpret the situation if the company manufactures and sells 100 tins in a day?
- A company produced a commodity with Rs. 10000 fixed costs. The variable costs are estimated to 25% of the total revenue received on selling the product at a rate of Rs. 6 per unit.

Find the total revenue, total cost and profit function.

22. A company produced a commodity with Rs. 20,000 fixed costs. The variable cost is estimated to 35% of the total revenue when it is sold at a rate of Rs. 6 per unit. Find the total revenue, total cost and profit functions of the revenue when it is sold.
23. The demand function is $x = \frac{24 - 2p}{3}$ where x is the number of units demanded and p is the price per unit. Find the revenue function R in terms of p .
24. A music-system manufacturer determines that its total cost for producing x units is given by $C(x) = 500x^2 + 4500x + 10,000$. Each unit can be marketed for Rs. 9000. Determine the break-even point.
25. ABC company Ltd. is planning to market a new model of shaving razor. The fixed cost of the company for the production of the razors is found to be Rs. 27,000 and the cost for the material and labour to produce each unit of razor is estimated to be Rs. 8 per unit. If each razor can be sold in the market for Rs. 12.00, find (i) the cost function for producing x razors, and (ii) the break-even point.
26. A company has fixed costs of Rs. 20,000 and the cost of producing one unit of their product is Rs. 5. If each unit is sold for Rs. 9, find the break-even point.
27. For the first year, the fixed cost for setting up a new electronic pocket calculators company is Rs. 300,000. The variable cost for producing a calculator is Rs. 70. The company expects the revenue from the sales of the calculator to be Rs. 270 per calculator. Find (i) the cost function (ii) the revenue function (iii) the break-even point, and (iv) the number of calculators produced for which the company will suffer a loss.
28. A company decides to set up a small production plant for manufacturing electronic clocks. The total cost for initial set up is Rs. 900,000. The additional cost for producing each clock is Rs. 300. Each clock is sold at Rs. 750. Find (i) the cost function (ii) the revenue function (iii) the profit function, and (iv) the break-even point.
29. A company sells its product for Rs. 4 per unit. Fixed costs for the company are Rs. 2800 and variable costs are estimated to run 30% of the total revenue. Determine (i) the total revenue function (ii) the total cost function (iii) the break-even point, and (iv) the quantity the company must sell to cover its fixed cost. ,
30. A text-book publisher finds that the production costs directly attribute to each book are Rs. 20 and that the fixed costs are Rs. 10,000. If each book can be sold for Rs. 30, determine (i) the cost function (ii) the revenue function, and (iii) the break-even point.
31. The pricing policy of a company follows the demand equation $p = D(x)$, $D(x)$ being the price per unit when x units are demanded. After studying the market trends the company determines the price function that is given by $D(x) = 2000 - 4x$. If the product is to be marketed the company will incur a fixed cost of Rs. 60,000 and will have to pay Rs. 600 for each unit that is produced and placed in the store. At what sales level can the company expect to recover its costs ?
32. $C(x) = 5x + 350$ and $R(x) = 50x - x^2$ are respectively the total cost and total revenue functions for a company that produces and sells x units of a particular product. Find: (i) the break-even values (ii) the values of x that produce profit (iii) the values of x that result in a loss.
33. The fixed cost of a new product is Rs. 18000 and the variable cost per unit is Rs. 550. If the demand function is $p(x) = 4000 - 150x$, find the break even values.
34. The fixed cost of a new product is Rs. 35000, and the variable cost per units is Rs. 500. If the demand function is $p(x) = 5000 - 100x$, find the break even values.
35. A company wants to launch a new product. It invested Rs. 37500 as fixed cost and Rs. 200 per unit as the variable cost of production. The revenue function for the sale of x units is given by $R(x) = 4825x - 125x^2$. Find the breakeven point (s).

36. A company starts producing pens and finds that the production cost of each pen is Rs. 10 and the fixed expenditure of production is Rs. 4500. If each pen is sold for Rs. 25, determine (i) cost function (ii) revenue function (iii) the breakeven point.
37. A company launched a new product with fixed cost of Rs. 25000 and the variable cost per unit is Rs. 1500. The revenue received on the sale of x units given by $8500x - 400x^2$. Find: (i) profit function (ii) break-even points.
38. The price of selling one unit of a product when x units are demanded is given by the equation $p = 4000 - 2x$. The fixed cost of product is Rs. 20000 and Rs. 1484 per unit are paid for the product to place in a store. Find the level of sales at which the company expect to cover its costs.
39. A company paid Rs. 16100 towards rent of the building and interest on loan. The cost of producing one unit of the item is Rs. 20. If each unit is sold for Rs. 27, find the breakeven point.
40. The fixed cost of a product is Rs. 20000 and cost of production per unit is Rs. 75. If each unit is sold for Rs. 100, find the break-even value. Also find the values of x for which the company always results in profit. ¹
41. A firm paid Rs. 25,000 as rent of its office and Rs. 15,200 as the interest of the loan taken to produce x units of a commodity. If the cost of production per unit is Rs. 8 and each item is sold at price of Rs. 75, find the profit function. Also, find the break-even point.
42. A calculator manufacturing company finds that the daily cost of procuring x calculators is given by $C(x) = 250x + 7500$.
- (i) If each calculator is sold for Rs. 350, find the maximum number of calculators that must be produced daily and sold to ensure no loss.
- (ii) If the selling price is increased by Rs. 250. What would be the break even point?
43. A computer software manufacturer wants to start the production of floppy disks. He observes that he will have to spend Rs. 2 lakhs for the technical know-how. The cost of setting up the machine is Rs. 88,000 and the cost of producing each unit is Rs. 30. He can sell each floppy at Rs. 45. Determine (i) the total cost function for producing x floppies, and (ii) the break-even point.
44. The fixed cost of a new product is Rs. 30,000, and the variable cost per unit is Rs. 800. If the demand function is $p(x) = 4500 - 100x$, find the break even values.
45. A manufacturer's total cost function is $C = 1500 + 30x + x^2$, find : (i) the average cost function, (ii) the marginal cost function, (iii) the marginal cost when 20 units are produced, and (iv) the actual cost of producing twenty first unit. Give the economic interpretation to these results.
46. If the total cost function for a manufacturer is given by $C = \frac{5x^2}{\sqrt{x^2+3}} + 5000$, find the marginal cost function.
47. The cost of producing x units of a product is given by $C = 7x + 130$. Show that the marginal cost is always constant.
48. The cost of producing and marketing x units of a certain commodity is given by $C = x^3 + 1500$. Find the marginal cost when (i) $x = 4$ (ii) $x = 8$.
49. If $C = 2x \left(\frac{x+4}{x+1} \right) + 6$ is the total cost of production of x units of a certain product, show that the marginal cost falls continuously as the output x increases.
50. If $C = ax \left(\frac{x+b}{x+c} \right)$, $b > c$ is the total cost function of producing x units of a certain commodity, show that average and marginal costs fall continuously as the output x increases.
51. If the total cost $C(x)$ of a firm is $C(x) = 0.005x^3 - 0.02x^2 - 30x + 5000$, where x is the output. Determine (i) average cost (ii) marginal cost.
52. Show that the slope of average cost curve is equal to $\frac{1}{x}(MC - AC)$ for the total cost function

$$C = ax^3 + bx^2 + cx + d.$$

53. The average cost function for a product is given by $AC = 0.0002x^3 - 0.05x + 7 + \frac{8000}{x}$, where x is the output. Find the marginal cost function. What is the marginal cost when 100 units are produced? Interpret your result.
54. The cost function of a firm is given by $C(x) = 2x^2 - 4x + 5$. Find (i) the average cost and, (ii) the marginal cost, when $x = 2$.
55. The cost function $C(x)$ of a firm is given by $C(x) = 3x^2 - 6x + 5$. Find: (i) the average cost and, (ii) the marginal cost, when $x = 2$.
56. The cost function $C(x)$ of a firm is given by $C(x) = 2x^2 - 4x + 5$. Find (i) the average cost, and (ii) the marginal cost, when $x = 10$.
57. Given the total cost function for x units of a commodity as $C(x) = \frac{x^3}{3} + x^2 - 15x + 3$. Find the (i) marginal cost function and (ii) average cost function.
58. The cost function of a firm is given by $C = 3x^2 - 2x + 3$. Find (i) the average cost and (ii) the marginal cost, when $x = 3$.
59. The average cost function associated with producing and marketing x units of an item is given by $AC = 2x - 11 + \frac{50}{x}$. Find the total cost function and marginal cost function. Also, find the range of output for which AC is increasing.
60. The total cost function for a production and marketing activity is given by $C(x) = \frac{3}{4}x^2 - 7x + 27$. Find the level of output (number of units produced) for which $MC = AC$.
61. The average cost of producing x units of a commodity is given by $AC = \frac{x^2}{200} - \frac{x}{50} - 30 + \frac{5000}{x}$. Find the marginal cost function and verify that $\frac{d}{dx}(AC) = \frac{MC-AC}{x}$.
62. If the total cost function is given by $C = 3 - 2x + 5x^2$, where x is the quantity of output, show that $\frac{d}{dx}(AC) = \frac{MC-AC}{x}$, where MC and AC are the marginal cost and average cost, respectively.
63. The total cost function is given by $C = x + 2x^3 - 3.5x^2$. Find the marginal average cost function (MAC). Also, find the points where the MC curve cuts the x -axis and y -axis.
64. If $C(x) = 0.05x - 0.2x^2 - 5$, find the level of output x for which the average cost AC becomes equal to the marginal cost.
65. If the total cost of producing and marketing x units of a certain commodity is given by $C(x) = 3a e^{-x/3}$. Verify that the shape of AC curve is given by $\frac{MC-AC}{x}$.
66. Given the total cost function for x units of a commodity as $C(x) = \frac{x^3}{3} + 3x^2 - 7x + 16$. Find (i) the marginal cost (ii) the average cost. Also, show that the marginal average cost is given by $\frac{xMC-C(x)}{x^2}$.
67. A firm has found from past experience that its profit in terms of number of units produced is given by $P(x) = -\frac{x^3}{3} + 729x - 2500$, $0 \leq x < 35$. Compute: (i) the value of x that maximize the profit, (ii) the profit per unit of the product when this maximum level is achieved.
68. A company finds that it can sell out a certain product that is produced at the rate of Rs. 2 per unit. It estimates the cost function to Rs. $\left\{1000 + \frac{1}{2}\left(\frac{x}{50}\right)^2\right\}$, for x units produced and sold. Find: (i) the expression for total profit, if x units are produced, (ii) the number of units produced that will maximize the profit, and (iii) the amount of maximum profit.
69. A manufacturer finds that his product can be assembled at a total cost $C = \text{Rs. } (200 + 30x)$, where x is the number of units manufactured. Assume that the price at which he can sell each

- unit is given by $p = \text{Rs.} \left(150 - \frac{x}{5}\right)$. What level of product will maximize the total profit ? What is the corresponding selling price per unit ? What is the total profit at this level of production?
70. A sitar manufacturer can sell x sitars per week at Rs. p each, where $5x = 375 - 3p$. The cost of production is Rs. $\left(500 + 13x + \frac{x^2}{5}\right)$. Find how many sitars should he manufacture for maximum profit and what is this profit?
71. If the total cost of a firm is $C(x) = \frac{x^3}{3} - 5x^2 + 30x + 15$, where C is the total cost and x is the output, and price under perfect competition is given by 6, find for what value of x the profit will be maximized?
72. A monopolist firm has the following cost and demand functions : $C = 50x + 3000$, $p = 100 - 0.01x$. Find out the profit maximizing output and price.
73. The demand and total cost function for a product facing a monopolist are given as: $p = 5(2-x)$ and $C = 10 + 3x^2 - 2x^3$ respectively. Determine the optimum level price and output for profit maximization.
74. A monopolist has a demand function $x = 106 - 2p$ and the average cost function $AC = 5 + \frac{x}{50}$, where p is the price per unit output and x is the number of units of output. If the total revenue is $R = p \cdot x$, determine the most profitable output and the maximum profit.
75. Find the profit maximizing output given the following revenue and cost functions: $R(x) = 1000x - 2x^2$, $C(x) = x^3 - 59x^2 + 1315x + 2000$.
76. A given product can be manufactured at a total cost $C = \text{Rs.} \left(\frac{x^2}{100} + 100x + 40\right)$, where x is the number of units produced. The price at which each unit can be sold is given by $p = \text{Rs.} \left(200 - \frac{x}{400}\right)$. Determine the production level x at which the profit is maximum. What is the price per unit and the total profit at this level of production?
77. The profit of a monopolist is given by $P(x) = \frac{8000x}{500+x} - x$. Find the value of x for which $P(x)$ is maximum. Also, find the maximum profit.
78. Suppose a manufacturer can sell x items per week at a price $p = 20 - 0.001x$ rupees each when it costs $y = 5x + 2000$ rupees to produce x items. Determine the number of items he should produce per week for maximum profit.
79. The demand function and total cost function of a monopolist are given as $p = 13 - 15x$ and $C = -x^3 + 3x^2$ respectively. At what level of price will there be maximum profit? What is the maximum profit?
80. Given the demand function $p = 21 - 4x$ and the average cost function $AC = 2$. Determine the profit maximizing output. What should be the impact of tax of Rs. t per unit of output on profit?
81. If the cost function and revenue function of x units of an item are given by $C(x) = x^3 - \frac{75}{2}x^2$ and $R(x) = 50x - x^2$. Find the number of items to be produced and sold to have maximum profit. Find the maximum profit also.
82. A radio manufacturer produces x sets per week and sells at Rs. p per set, where $x = \frac{600-p}{8}$. The cost of production of x sets is Rs. $x^2 + 78x + 2000$. Find the number of sets to be produced and sold per week so as to have maximum profit. Calculate the maximum profit also.

ANSWERS

17. (i) $R = 85x - x^2$ (ii) $P = 83x - x^2 - 75$
 18. (i) $p = 954$, $R = 1908$ (ii) $p = 900$, $R = 4500$

19. $x = \frac{5}{3}p - 10$, $R(x) = 6x + \frac{3x^2}{5}$, $R = 1800$
 20. $P(x) = 3x - 600$, $P(500) = \text{Rs. } 900$, company incurs a loss of Rs.300
 21. $R(x) = 6x$; $C(x) = 10000 + \frac{3}{2}x$, $P(x) = \frac{9}{2}x - 10000$
 22. $R(x) = 6x$, $C(x) = 20000 + \frac{21}{10}x$, $P(x) = \frac{39}{10}x - 20,000$ 23. $R = 8p - \frac{2}{3}p^2$
 24. $x = 4, 5$ 25. (i) $C(x) = 27,000 + 8x$ (ii) 6750 26. $x = 5,000$
 27. (i) $C(x) = 70x + 300,000$, (ii) $R(x) = 270x$ (iii) $x = 1500$
 (iv) $x < 1500$ 28. (i) $C(x) = 900,000 + 300x$ (ii) $R(x) = 750x$
 (iii) $450x - 900,000$ (iv) 2000
 29. (i) $R(x) = 4x$ (ii) $C(x) = 2800 + \frac{6}{5}x$ (iii) 1000 (iv) 700
 30. (i) $C(x) = 20x + 10,000$ (ii) $R(x) = 30x$ (iii) $x = 1000$
 31. 50, 300 32. (i) 10, 35 (ii) $10 \leq x \leq 35$
 (iii) $x > 35$ or $x < 10$ 33. 8, 15 34. 10, 35 35. 12, 25
 36. (i) $C(x) = 4500 + 10x$, (ii) $R(x) = 25x$, (iii) 300
 37. (i) $P(x) = 7000x - 400x^2 - 2500$, (ii) 5, 12.5 38. 1250, 8
 39. 2300 40. 800, $x > 800$ 41. $P(x) = 67x - 40$, 200, 600
 42. (i) 75 (ii) 30
 43. (i) $C(x) = 288,000 + 30x$ (ii) 19,200 44. $x = 12, 25$
 45. (i) $AC = \frac{1500}{x} + 30 + x$ (ii) $MC = 30 + 2x$ (iii) $MC = 70$ (iv) 71
 46. $MC = \frac{5x(x^2+6)}{(x^2+3)^2}$ 48. (i) 48 (ii) 192
 51. (i) $AC = 0.005x^2 - 0.02x - 30 + \frac{15000}{x}$ (ii) $MC = 0.015x^2 - 0.04x - 30$
 53. $MC = 0.0008x^3 - 0.10x + 7$, $MC = 797$
 54. (i) 2.5 (ii) 4 55. (i) 2.5 (ii) 6 56. (i) 16.5 (ii) 36
 57. (i) $x^2 + 2x - 15$ (ii) $\frac{x^2}{3} + x - 15 + \frac{3}{x}$ 58. (i) 8 (ii) 16
 59. $C(x) = 2x^2 - 11x + 50$, $MC = 4x - 11$, $0 \leq x \leq 5$ 60. 6 units
 61. $MC = \frac{3x^2}{200} - \frac{x}{25} - 30$ 63. $4x - 35$, (0,1), (1, 0), $(\frac{1}{6}, 0)$
 64. 5 66. (i) $MC = x^2 + 6x - 7$ (ii) $AC = \frac{x^2}{3} + 3x - 7$
 67. (i) 27, (ii) Rs. 393.40 68. (ii) 5000 (iii) Rs. 4000
 69. (i) Rs. 300 (ii) Rs. 90 per unit (iii) Rs. 17,800
 70. $x = 30$, Max. profit = Rs. 1180 71. 6 72. 2500, $p = 75$
 73. $p = 5$, $x = 1$ 74. 46 units, Profit = 1107.68 75. 35 units
 76. $x = 4000$ unit, price = Rs. 190, profit = Rs. 199,960 77. 1500, Rs. 4500 78. 7500
 79. 1, Rs. 6 80. $x = \frac{19}{8}$ units, $x = \frac{19-t}{8}$ unit 81. 25 units, Profit = Rs. 8437.50
 82. 29 sets, Rs. 5,569